Analysis of Building's Response to Diurnal
Variation in Ambient Temperature

by

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Submitted to the Department of Mechanical Engineering in Partial Fulfillment of the
Requirements for the Degree of

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ABSTRACT

An analysis is conducted to determine the number of nodes necessary to understand the influence that ambient temperature has on a building's interior air temperature. The simple case of a wall of homogeneous material is modeled as an electrical system: the building walls are modeled as resistors capacitors, the ambient temperature as the sinusoidal input function, and the inside air temperature as the output function. An analytical solution is obtained using differential equations. Numerical examples for various building materials and a range of thicknesses are explored using Matlab™. Of particular interest is the time delay and amplitude until the temperature of interior air peaks. The number of nodes that best describes a system is dependent on the type of material and thickness of the wall. For high-density materials such as concrete, time delays will be less than lower-density materials because delay depends on the thermal resistance and capacitance. The thicker the wall the more delay there is for the interior air to peak. There are greater increases in time delay with an increase in wall thickness and density. Polystyrene and wood had longer time delays than concrete and brick, with concrete having the overall lowest amount of phase shift and polystyrene having the longest phase shift. For materials with higher densities, one node would be enough for wall thicknesses of 3-6 inches and two nodes for wall thickness of 10-14 inches. For better insulating material with lower densities, two nodes are appropriate for thicknesses of 3-6 inches while 10-14 inches would be best described with three nodes.

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Introduction:

Ambient temperature varies throughout the course of a 24 hour period. A paramount research goal has been to further understand the effects this variation has on a building's interior temperature. Such understanding provides better input to models of a building's interior thermal mass, such as floor slabs. Better modeling leads ultimately to reduction in energy consumption.

The response of buildings to thermal stimuli lends itself well to a "nodal" modeling approach. This approach serves as a nice compromise between precision and simplicity. But how many nodes are necessary to precisely analyze, for example, a building/wall system? This study addresses the trade-off between model complexity and precision in an informal way by varying the number of nodes used to model a building-wall system. A critical characteristic of the system's response, its time-delay, is analyzed for a variety of materials.

Theory:

Time-lag:

Ambient temperature is modeled as a sinusoidal input. Due to the resistance of the wall, a time delay is expected between the peak ambient temperature and the peak interior temperature. The magnitude of the delay is determined by the thickness, density, and specific heat of the wall material. Larger time-lags indicate that it will take longer for the temperature of air inside the building to respond to the outside air temperature. For example, if the time-delay is x amount of hours and the outside air temperature peaks at 3 pm, then the inside air temperature will peak x many hours after 3 pm. In general, the peak temperature of the inside air will be much less than the outside temperature because of the resistance and heat capacity of the walls. The damping of the temperature is also a function of the time constant of the system.

Capacitance of walls:

The heat capacitance of the wall is determined by the material of which it is composed. Concrete, Wood, Brick, and Polystyrene (poly) are the chosen materials because they are a representative set of construction materials. Polystyrene and wood have higher heat capacities because of their molecular composition. Although polystyrene and wood have higher heat capacities than concrete and brick, they are not good resistors solely due to the high heat capacities. Wood has a higher specific heat than polystyrene yet it is a worse insulator than polystyrene.

Thermal Conductivity

The materials chosen vary in their thermal conductivity: the masonry type materials are good thermal conductors, whereas wood and polystyrene are very poor thermal conductors. Materials with low thermal conductivity will be good resistors to heat; they will minimize the quantity of heat that passes, in unit time, through the walls. As resistance increases, thermal conductivity will decrease, and, as thermal conductance increases, there will be an increase in density. Thus, we have that:

$$K \propto \frac{1}{R} \text{ and also } k \propto \rho \qquad (1)$$

Where K = thermal conductivity, R = Resistance through walls, ρ = density.

Amplitude:

The amplitude of the system output is proportional to the peak time delay. As the amplitude of the system's response decreases, the time needed for the air inside the building to reach its peak temperature will increase; this is shown in equation 2. If the time for the air inside the building to peak is a short phase, then the temperature of the air will be slightly less than the outside temperature.

$$T_{amppeak} \propto \frac{1}{tpeak} \qquad (2)$$

where,

$T_{amppeak}$ is the amplitude of the inside air temperature when it peaks. Amplitude of the system output is also dependent on the geometry of the wall. If the thickness of the wall increases then it will take longer for the temperature from the air outside to travel through the wall and reach the inside air.

$$T_{amppeak} \propto \frac{1}{thicknessofwall} \qquad (3)$$

Peak:

The peak time delay of the system output, t_{peak}, is proportional to the resistance of the wall:

$$t_{peak} \propto \sum Rw \qquad (4)$$

Thicker walls imply that it will take longer for the system output to peak. Resistance is inversely proportional to thermal conductivity of the wall. As evident in equation (5) below, if the wall material's thermal conductivity is high, then its ability to resist is diminished:

$$R_{con} = \frac{t_w}{KA} \qquad (5)$$

Where the resistance due to conduction is R_{con}, t_w is the thickness of the wall, and A is the surface area of the building, and K is the thermal resistance.

Methodology:

Method 1: Analytic Solution for 4 Cases

The following analysis is of the simplest form: only boundary layer resistances are considered. The system is a building with no windows, doors, airflow, leaks, or radiation. Developing electrical analogies facilitates the analysis. See appendix for in depth solving process.

Case 1: Single Node

In this case, the wall is modeled as a single node. The formulation consists of two differential equations, both of which are first order. The temperature in the middle of the wall (where $t=t_w/2$) is unknown, but by combining the two differential equations, one generates an additional differential equation that relates the ambient temperature and the interior temperature: $T_{a,o}$ and $T_{a,i}$. This case will have two equal resistances due to conduction in the walls. The differential equations are as follows:

$$C_a \frac{dT_{a,i}}{dt} = \frac{T_w - T_{a,i}}{R_3 + R_4} \quad \rightarrow \quad T_{au1} \frac{dT_{a,i}}{dt} + T_{a,i} = T_w \tag{6}$$

$$T_{au1} = (R_3 + R_4)C_a \tag{6a}$$

$$C_w \frac{dT_w}{dt} = \frac{T_{a,o} - T_w}{R_1 + R_2} + \frac{T_{a,i} - T_w}{R_3 + R_4} \quad \rightarrow \quad T_{au2} \frac{dT_w}{dt} + T_w = PT_{a,o} + NT_{a,i} \tag{7}$$

$$T_{au2} = \left[\frac{(R_1 + R_2)(R_3 + R_4)}{2R + R_1 + R_4}\right] C_w \tag{7a}$$

where,

T_{au1} is the time constant of the system that describes the second half of the wall that is closest to the interior air temperature. T_{au2} is the time constant of the system composed of the half of the wall that is closest to the ambient temperature. T_w is the temperature of the wall at one node and the capacitance of the wall and air is represented by their respective C.

The solution of our system output is in transfer function form shown below:

$$T_{a,i}(s) = \frac{PT_{a,o}(s)}{(T_{au1}T_{au2})s^2 + (T_{au1} + T_{au2})s + (1-N)} \tag{8}$$

The system input is a sinusoidal described as $T_{a,o}$=Asin(wt). Solutions and nomenclature can be found in the appendix for all cases.

Case 2: Two Nodes

In case two, the wall-model has two nodes. Each node will have half the total capacitance, and the wall temperatures for nodes one and two will be T_{w1} and T_{w2} respectively. There will be 3 differential equations that, when combined, result in one third-order differential equation in terms of the ambient and interior temperatures. In this case there will be four equal resistances due to conduction in the wall. The following are the differential equations that describe the two-node model:

$$C_a \frac{dT_{a,i}}{dt} = \frac{T_{w2} - T_{a,i}}{R_5 + R_6} \tag{9}$$

$$\frac{C_w}{2}\frac{dT_{w2}}{dt} = \frac{T_{w1} - T_{w2}}{R_3 + R_4} + \frac{T_{a,i} - T_{w2}}{R_5 + R_6} \tag{10}$$

$$\frac{C_w}{2}\frac{dT_{w1}}{dt} = \frac{T_{a,o} - T_{w1}}{R_1 + R_2} + \frac{T_{w2} - T_{w1}}{R_3 + R_4} \tag{11}$$

Case 3: Combination of Concrete and Polystyrene

In case 3, the wall consists of two different types of materials: one is masonry, and the other is an insulating material. Concrete and polystyrene are chosen as a well-suited combination for insulation purposes. The following three equations describe case 3.

$$C_a \frac{dT_{a,i}}{dt} = \frac{T_2 - T_{a,i}}{R_5 + R_6} \tag{13}$$

$$C_2 \frac{dT_2}{dt} = \frac{T_1 - T_2}{R_3 + R_4} + \frac{T_{a,i} - T_2}{R_5 + R_6} \qquad (14)$$

$$C_1 \frac{dT_1}{dt} = \frac{T_{a,o} - T_1}{R_1 + R_2} + \frac{T_2 - T_1}{R_3 + R_4} \qquad (15)$$

Case 4: Three Nodes

Case four describes more accurately what is going on inside the wall. The following 4 differential equations are combined to give one differential equation in terms of the temperature of air outside and the temperature of air inside the building. A transfer function is not analytically solved for case 4, instead it is solved using MATLAB™.

$$C_a \frac{dT_{a,i}}{dt} = \frac{T_3 - T_{a,i}}{R_7 + R_8} \qquad (16)$$

$$\frac{C}{3}\frac{dT_3}{dt} = \frac{T_{a,i} - T_3}{R_7 + R_8} + \frac{T_2 - T_3}{R_5 + R_6} \qquad (17)$$

$$\frac{C}{3}\frac{dT_2}{dt} = \frac{T_1 - T_2}{R_3 + R_4} + \frac{T_3 - T_2}{R_5 + R_6} \qquad (18)$$

$$\frac{C}{3}\frac{dT_1}{dt} = \frac{T_2 - T_1}{R_3 + R_4} + \frac{T_{a,o} - T_1}{R_1 + R_2} \qquad (19)$$

Method 2: State Space Form in Matlab

In method two, the differential equations that described all the cases were written in matrix form and programmed in Matlab (see appendix for source code). Matlab software solved the differential equations and yielded the same results as the analytic solution. Matlab's utility is evident when dealing with more than 3 differential equations. See appendix for values of A, B, C, D, E, M, N, O, P, Q, and S.

Case 1: 1 Node

$$\frac{d}{dt}\begin{bmatrix} T_{a,i} \\ T_w \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_{au2}} & \frac{1}{T_{au1}} \\ \frac{P}{T_{au2}} & -\frac{1}{T_{au2}} \end{bmatrix} \begin{bmatrix} T_{a,i} \\ T_w \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{N}{T_{au2}} \end{bmatrix} T_{a,o} \qquad (20)$$

Case 2: 2 Nodes

$$\frac{d}{dt}\begin{bmatrix} T_{a,i} \\ T_{w2} \\ T_{w1} \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_{au1}} & \frac{1}{T_{au1}} & 0 \\ \frac{P}{T_{au2}} & -\frac{1}{T_{au2}} & \frac{N}{T_{au2}} \\ 0 & \frac{S}{T_{au3}} & -\frac{1}{T_{au3}} \end{bmatrix} \begin{bmatrix} T_{a,i} \\ T_{w2} \\ T_{w1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{Q}{T_{au3}} \end{bmatrix} T_{a,o} \quad (21)$$

Case 3: Combination of Concrete and Polystyrene

$$\frac{d}{dt}\begin{bmatrix} T_{a,i} \\ T_{2} \\ T_{1} \end{bmatrix} = \begin{bmatrix} -A\frac{1}{C_a} & A\frac{1}{C_a} & 0 \\ A\frac{1}{C_2} & -D\frac{1}{C_2} & B\frac{1}{C_2} \\ 0 & E\frac{1}{C_1} & -C\frac{1}{C_1} \end{bmatrix} \begin{bmatrix} T_{a,i} \\ T_{2} \\ T_{1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ C\frac{1}{C_1} \end{bmatrix} T_{a,o} \quad (22)$$

Case 4:

$$\frac{d}{dt}\begin{bmatrix} T_{a,i} \\ T_{3} \\ T_{2} \\ T_{1} \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_{au1}} & \frac{1}{T_{au1}} & 0 & 0 \\ \frac{1}{T_{au2}P} & -\frac{1}{T_{au2}} & \frac{1}{T_{au2}N} & 0 \\ 0 & \frac{1}{T_{au3}X} & -\frac{1}{T_{au3}} & \frac{1}{T_{au3}Q} \\ 0 & 0 & \frac{M}{T_{au4}} & -\frac{1}{T_{au4}} \end{bmatrix} \begin{bmatrix} T_{a,i} \\ T_{3} \\ T_{2} \\ T_{1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{S}{T_{au4}} \end{bmatrix} T_{a,o} \quad (23)$$

Results:

Specific Heat, Density, and Thermal Conductivity

Materials with less mass require less energy to increase their temperature. This is illustrated below in figure 1 for various materials of interest. Figure 1 shows the variation in specific heat and density for the materials tested. Concrete has the lowest specific heat and has the highest density of the four materials. The material with the lowest density is wood which also has the highest specific heat. Although one may expect that the higher the specific heat the more thermal resistance it will provide, but this is not always the case as will be seen in the data analysis. Polystyrene will actually be the materials with the highest thermal resistance of the four materials.

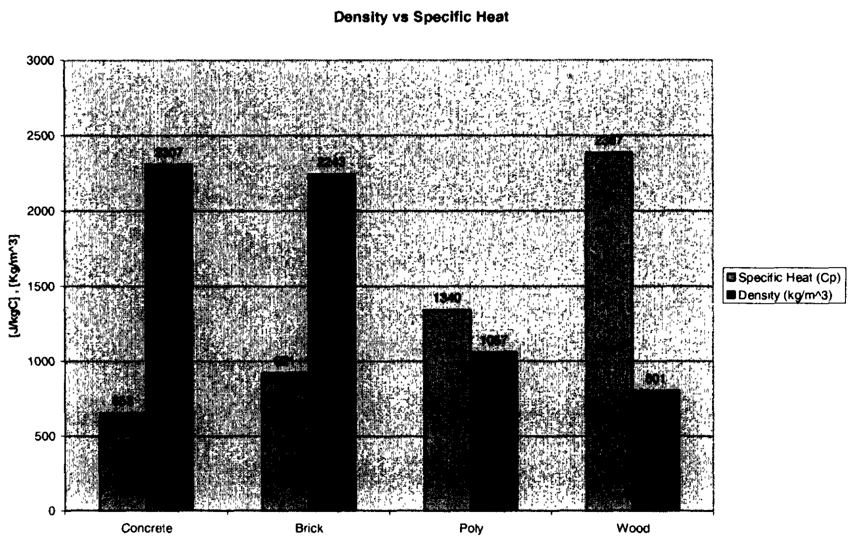


Figure 1:Density vs. Specific Heat of the materials

Thermal conductivity is also a function of density. With an increase in density, there will also be an increase in thermal conductance. A material with a low density will be a poor thermal conductor, and, as a result, the resistance of the material will increase. A less dense wall will be unable to resist against the influence of outside temperatures, and the result will be a minimal difference in outside and inside temperature.

As shown in figure 2, the masonry materials such as concrete and brick, which have high densities, will also have the highest thermal conductance. Because concrete and brick will have less resistance, the time until the interior air temperature reaches its peak will be less than that of less dense materials like polystyrene and wood. Polystyrene and wood have lower densities, higher specific heats, and lower thermal conductivity. Essentially, polystyrene and wood are better resistors and this will lead to a longer delay for inside air to reach its peak.

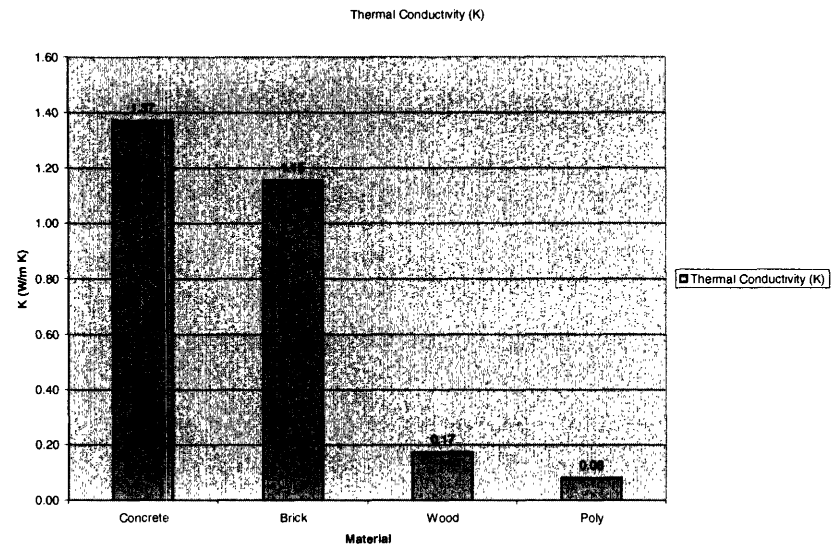


Figure 2:Thermal Conductivity of the Materials

Phase Shifts and Amplitudes:

The temperature of the air outside travels through the wall and eventually reaches the temperature of the air inside. The phase shift is the amount of time it takes for the temperature of air outside to reach the inside air. If the outside air reaches its peak temperature at 1pm and the phase shift is 3 hours, then the temperature of the air inside the building will reach its peak temperature at 4 pm. Due to the resistance of the wall there will be a difference in the peak temperature between outside and inside air. The air inside will have a lower temperature (amplitude) because it will be lost through the wall and depending on the wall material that amplitude will vary. The amplitude (in units of Celsius) of the average ambient temperature is 7.5 degrees Celsius, the amplitude of the materials will be less than 7.5 [C]. Materials with higher thermal conductivities, such as concrete and brick, will have higher amplitudes. Materials that are good resistors, such as polystyrene, that have low thermal conductance will have significantly lower amplitudes therefore greater difference in outside and inside air.

Thicker walls have greater phase shifts and magnitudes. One might wonder how this trend is affected by increasing the number of nodes in the model. It appears that, for a given percentage change in wall thickness, models with more nodes lead to (different, greater-than-proportional, exaggerated) percentage changes in phase shifts and magnitudes. There will be cases where modeling a wall as a three-node model will be crucial but there will also be cases where modeling it as a two- or three-node model will not make an immense difference. The frequency graphs show that, for low frequencies, the amplitude of the three-node case will be slightly higher than both one- and two-node cases. At a certain level of frequency, the three-node case has amplitude that is in between the other two (see appendix). Tables 1 and 2 below summarize the effect of node-number and thickness on phase-shifting. Table 1 and Table 2 show the difference in phase shift between the nodes for a thickness of 3, 6, 10, and 14 inches; where Table 1 indicates the materials with lowest phase shift and Table 2 indicates the materials with the highest phase shift depending on the thickness and node difference.

Table 1: Summary of Material with Lowest amount of Phase shift

Lowest Phase Shift	**3 inches**	**6 inches**	**10 inches**	**14 inches**
Node 2 - Node 1	Concrete--0	Concrete- 50 mins	Concrete- 1 hr and 50 mins	Conrete- 3 hrs and 36 mins
Node 3 - Node 2	Concrete--0	Concrete- 5 mins	Concrete- 55 mins and Brick 56 mins	Concrete- 1 hrs and 12 mins
Node 3 - Node 1	Concrete--0	Concrete- 55 mins	Concrete- 2 hrs and 46 mins	Concrete- 4 hrs and 48 mins

Table 2: Summary of Material with Largest amount of Phase shift

Largest Phase Shift	**3 inches**	**6 inches**	**10 inches**	**14 inches**
Node 2 - Node 1	Poly- 1 hr and 56 mins	Poly/Wood - 4 hrs and 37 mins	Wood - 4hrs and 50 mins	Poly/Wood- 5 hrs
Node 3 - Node 2	Poly- 41 mins	Poly- 2 hrs and 32 mins	Poly- 4 hrs	Poly/ Wood- 5 hrs and 20 mins
Node 3 - Node 1	Poly- 2 hrs and 38 mins	Poly - 7 hrs	Wood/Poly- 8 hrs and 40 mins	Poly/Wood- 10 hrs and 24 mins

If a wall is composed of concrete the time for the interior air to reach its peak will be sooner than a wall composed of polystyrene, for a thickness of 3,6,10, and 14 inches. Polystyrene and wood however come very close to having the same amount of time delay for higher thicknesses.

Table 3 summarizes the amount of nodes necessary to describe the system.

Table 3: Amount of nodes necessary for given material and wall thickness

	1-node	2-nodes	3-nodes
Concrete	3 to 6 inch wall	10 inch wall	14 inch wall
Brick	3 to 10 inch wall	14 inch wall	
Polystyrene		3 to 6 inch wall	10 to 14 inch wall
Wood	3 inch wall	6 to 10 inch wall	14 inch wall

Table 4: Amplitude for 1 node

1 Node	0.075	0.15	0.2	0.25	0.3	0.35
Concrete	5.04	3.13	2.38	1.86	1.50	1.23
Brick	4.26	2.39	1.75	1.33	1.05	0.85
Wood	2.55	0.95	0.59	0.40	0.29	0.22
Poly	2.06	0.68	0.41	0.27	0.19	0.14

Table 5: Amplitude for 2 nodes

2 Nodes	0.075	0.15	0.2	0.25	0.3	0.35
Concrete	5.14	3.23	2.42	1.83	1.40	1.08
Brick	4.37	2.44	1.71	1.21	0.87	0.64
Wood	2.58	0.72	0.35	0.18	0.10	0.06
Poly	2.00	0.44	0.19	0.09	0.05	0.03

Table 6: Amplitude for 3 nodes

3 Nodes	0.075	0.15	0.2	0.25	0.3	0.35
Concrete	5.161	3.27	2.466	1.881	1.446	1.118
Brick	4.394	2.486	1.752	1.25	0.9007	0.6559
Wood	2.659	0.7358	0.3375	0.1667	0.08577	0.04522
Poly	2.076	0.4334	0.1747	0.07513	0.03378	0.01626

Table 7: Phase shift for each material with varying thickness and node in hours

Brick	t= 3	t= 6	t=10	t=14
Node 1	4.15	5.08	5.77	5.93
Node 2	5.77	6.69	8.77	10.15
Node 3	5.81	6.87	9.69	11.77

Polystyrene	t= 3	t= 6	t=10	t=14
Node 1	5.91	6.46	7.38	7.38
Node 2	7.85	11.08	12.00	12.46
Node 3	8.54	13.62	16.13	17.77

Wood	t= 3	t= 6	t=10	t=14
Node 1	5.08	6.00	6.69	6.69
Node 2	6.92	10.61	11.54	11.77
Node 3	7.39	12.69	15.46	17.08

Concrete	t= 3	t= 6	t=10	t=14
Node 1	3.69	5.08	5.54	6.46
Node 2	3.69	5.91	7.39	11.08
Node 3	3.69	6.00	8.31	15.69

Table 8: Time delay difference between nodes and thicknesses

Brick	t= 3	t= 6	t=10	t=14
Node 2 - Node 1	1.61	1.62	3.00	4.22
Node 3 - Node 2	0.05	0.17	0.93	1.61
Node 3 - Node 1	1.66	1.79	3.93	5.83

Polystyrene	t= 3	t= 6	t=10	t=14
Node 2 - Node 1	1.94	4.62	4.62	5.08
Node 3 - Node 2	0.69	2.54	4.13	5.31
Node 3 - Node 1	2.63	7.15	8.75	10.39

Wood	t= 3	t= 6	t=10	t=14
Node 2 - Node 1	1.84	4.61	4.85	5.07
Node 3 - Node 2	0.47	2.08	3.92	5.31
Node 3 - Node 1	2.31	6.69	8.77	10.39

Concrete	t= 3	t= 6	t=10	t=14
Node 2 - Node 1	0.00	0.83	1.85	4.62
Node 3 - Node 2	0.00	0.09	0.92	4.62
Node 3 - Node 1	0.00	0.92	2.77	9.23

Refer to Tables 4, 5, 6, 7 and 8 for the phase and magnitude analysis that follows.

Brick

Magnitude:

For thicknesses between three and six inches, the amplitudes determined by the models are the same for a frequency of the 7.268E-5 (rads/sec), which is the frequency of one day where frequency is equal to radians divided by 24 hrs converted into seconds. At 10 inches, the three-node model is slightly higher than the one-node. At 14 inches, the amplitude difference between the one- and three-node models is still slight, and between two- and three- nodes there is no noticeable difference. At low frequencies the magnitude difference is zero which does not help conclude which nodal approach is best.

Phase:

3 inches:

There is a slight difference in phase shift between the models. At a frequency of 7.268E-5 [rads/sec], the phase shift between two and three nodes is negligible. The phase shift between two- and one- node as well as three- and one- node is about 24.24 degrees (where 15 degrees equals an hour) which constitutes about an hour and thirty minutes. If modeling a 3 inch wall, one-node will suffice.

6 inches:

There is a bigger difference in phase shift between one-node with two- and three- nodes. This difference is higher than the 3 inch case. Since the phase shift between two- and three- nodes is only about 10 minutes, modeling the 6 inch case with three-nodes will not be necessary. The phase shift between one - and two- nodes will matter. Although slightly larger than the case of 3 inches, the phase shift is approximately an hour and thirty minutes. Since the magnitude experiences no change at a frequency of 7.268E-5 [rads/sec] then modeling it as one-node will not be implausible.

10 inches:

The 10 inch wall experiences larger phase shifts: (approximately 1.85 times that of a 6-inch brick wall). The phase shift between one- and two- nodes is three times that of two- and three- nodes. Modeling a 10 inch brick wall would be enough with two-nodes.

14 inches:

Phase shift between one- and two- nodes for 14 inch wall is about 4 hours, which is 1.4 times lager than that of the 10 inch wall. The phase shift between two- and three- nodes for a 14 inch wall is about an hour and 40 minutes, which is 1.74 times larger than the 10 inch wall. Between one- and three- nodes there is significant phase shift of about 5 hour and 50 minutes. Modeling a 14 inch wall with one-node will not be sufficient. The three-node model can be considered but a two-node model may be the better option.

Polystyrene

Magnitude:

3 inch:

The magnitude shows little to no difference between the nodes. The phase shift will be the determining factor for the nodal approach.

6 inch:

The magnitude at a frequency of 7.268E-5 [rads/sec] shows that one-node is slightly higher than that of two- and thee- nodes. Two- and three- node models have negligible difference. Modeling poly as one-node is not reasonable, but modeling it as two- or three- nodes would be determined by frequency analysis which will be determined in the phase analysis.

10 inch:

The magnitude difference between one-node and two- three- nodes is larger for the 10 inch wall than the 6 inch wall as expected. The 10 inch wall is showing a greater difference between two- and three- nodes, but still not enough to be solely the determining factor as to which should be the nodal model.

14 inch:

The magnitude difference between two- and three- nodes seems to have increase by a small increment from the 10 inch wall to the 14 inch wall. Perhaps the magnitude difference between two- and three- nodes will not increase significantly with the increase of wall thickness. However, there is a noticeable increase in the difference between one- and two- nodes as well as with one- and three- nodes. One-node will not be enough to analyze the wall but two- or three- nodes is still to be determined with the frequency analysis.

Phase:

3 inches:

Polystyrene does demonstrate some difference in magnitude between nodes but it is of negligible amount. In terms of phase shift, there is a considerable change from one- to two- nodes, even three-nodes. Between two- and three- nodes there isn't a great change, only about 41 minutes of phase shift. However, the shift between one- and two- nodes is about 2 hours, almost three times that of the phase shift between two- and three- nodes. Modeling a 3 inch wall as one-node is not recommended because two-nodes has two more hours of phase shift.

6 inches:

The phase shift from one- to two- nodes is about 2.4 times larger for the 6 inch case than the 3 inch case which translates to about 2 hours and 40 minutes. The shift from one- to two- nodes for a 6 inch wall is 4 hours and 36 minutes; where as the shift from two- to three- nodes is two hours and 30 minutes. Between one- and three- nodes there is a about a seven hour shift. A seven hour phase shift is substantial; doing a two-nodal model is not improbable.

10 inches:

For a wall of 10 inches, a two-nodal model is not enough. The phase shift between two- and three- nodes is 4 hours. Between one- and three- nodes there is an 8 hours phase shift.

14 inches:

As would be expected, a 14 inch wall will have the greatest phase and magnitude change. The phase shift between two- and three- nodes is about 5 hours and 20 minutes, where as one- and three- nodes it is about twice of the phase shift of the two- and three- nodes. The 14 inch wall would require a three-nodal model.

Wood

Magnitude:

3 inch:

There is no change in magnitude between the three nodes. The nodal approach will be determined by the phase shift.

6 inch:

There is a slight difference between one-node with two- or three- nodes. However, there is no difference between two- and three- nodes. The nodal model will either consists of two- or three- nodes which will be determined by the phase shift.

10 inch:

There is a greater change between one-node with two- or three- nodes, which eliminates one-node as a nodal model. Although there is a difference in magnitude between two- and three- nodes, it is a slight change that can't solely determined the model to choose. The phase shift will again be the determining factor.

14 inch:

The changes in magnitude from a 10 inch to a 14 inch are not significant for they are small changes. However, it is a significant difference from a 3 inch wall which showed little to no change in magnitude. The magnitude is much less for a 14 inch wall than a 3 inch wall as is expected. The magnitude alone cannot determine which nodal model to choose.

Phase:

3 inch:

A three-node model is not necessary for this wall thickness because the phase shift between two- and three- nodes is about 30 minutes. The phase shift between one- and two- nodes is about an hour and 20 minutes longer. A one-node model may suffice.

6 inch:

Seeing as there was no difference between magnitudes between two- and three- nodes, the phase shift will be the determining factor. The change between one- and two- nodes is 4 hours and 30 minutes where are one- to three- nodes it is about 6 hours. Therefore between two- and three- nodes there is only about 2 hours. One can model this as a two- or three- nodal model as long as a two hour phase shift is not a major concern.

10 inch:

Phase shifts for a wall of this thickness are considerably large. Between a one- and three- node there are 8 hours and 45 minutes of phase shift and half of that for one- to two- node. A three nodal approach is recommended for best description of system response.

14 inch:

The phase shift between 1 node and 3 nodes is about 10 hours where are 1 node and 2 nodes is 5 hours. A third node would give best description.

Concrete

Magnitude:

There is no change in magnitude with increase in nodes for wall thicknesses of 3, 6, and 10 inches. However there are changes in phase shift within nodes with increase in wall thickness.

Phase:

3 inch:

There is no phase shift between the nodes. One-node will give the same result as a three-node model. There is minimal magnitude change even for the case with 14 inches.

6 inch:

Although the phase shift is minimal, the shift from one- to two- nodes is a little less than an hour. The phase shift between two- and three- nodes is only 5 minutes, a negligible amount. Either a one- or two- node model will be a reasonable approach.

10 inch:

Since the phase shift from two- to three- nodes is less than an hour, then it seems unnecessary to proceed with a three-node model.

14 inch:

A three-node approach would be recommended because the phase shift from one- to three- nodes is twice that of one- to two- nodes. About 9 hours of phase change from one- to three- nodes.

Amplitudes

In our results for amplitudes, concrete and brick have amplitudes that increase when going from one- to two- nodes, see appendix for more data. For the case of a 6 in thick wall, wood's and polystyrene's amplitude decreases from one- to two- nodes however, the amplitude for concrete and brick increases instead, as seen in figure 3. Although the increase from one-node to two-nodes is not a large amount, only 3.2% and 1.7% for concrete and brick respectively, the expectation was that the amplitude for two nodes would be less due to the longer peak time delay. Why was the material behaving as such?

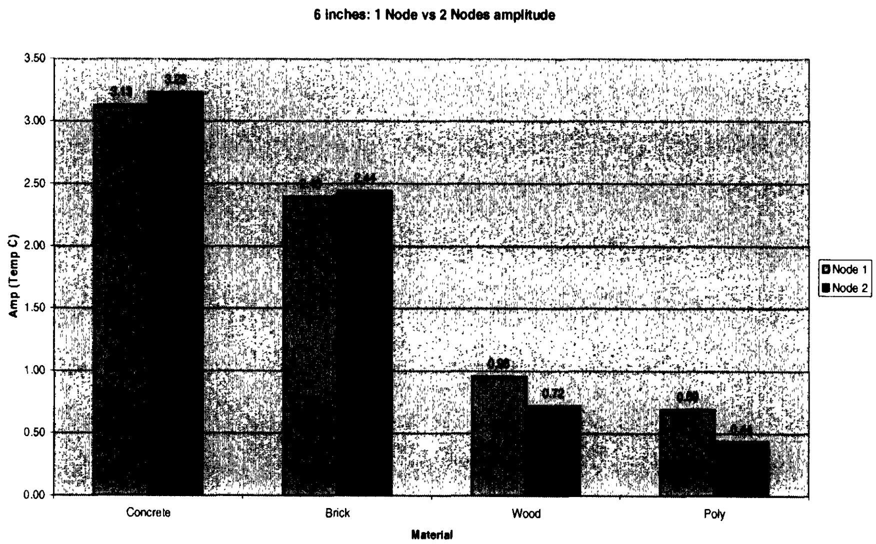


Figure 3: Amplitude of 1 Node vs. 2 Nodes for 6 in wall

Further testing was done by varying the thicknesses of the walls between 3 and 20 inches for both concrete and brick. The results for concrete are shown in figure 4 and those for brick are in figure 5. The amplitude of concrete for the case of two nodes is greater than the amplitude of the one-node case until it gets to around .25m (10inches), at 10 inches the amplitude for two-nodes is less than the amplitude for one-node and it continues to decrease with increase in wall thickness.

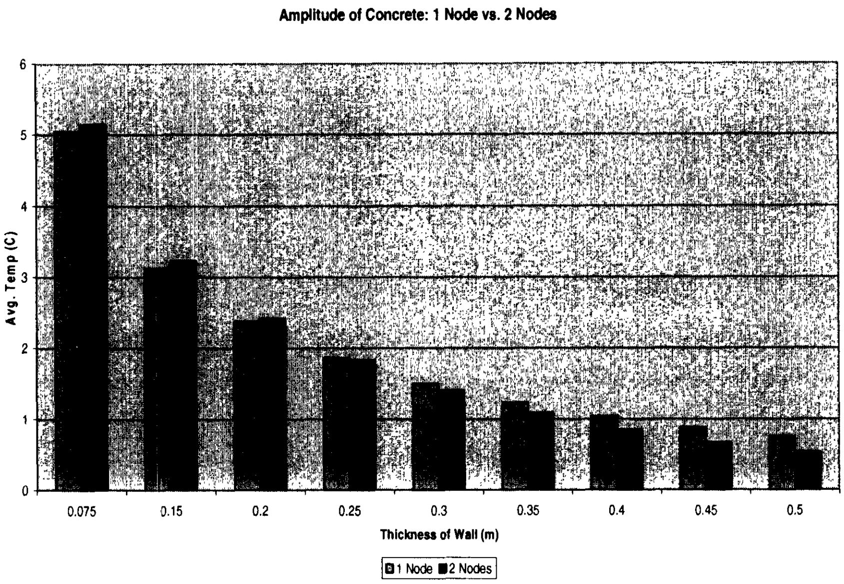


Figure 4:Amplitude of Concrete with varying wall thicknesses

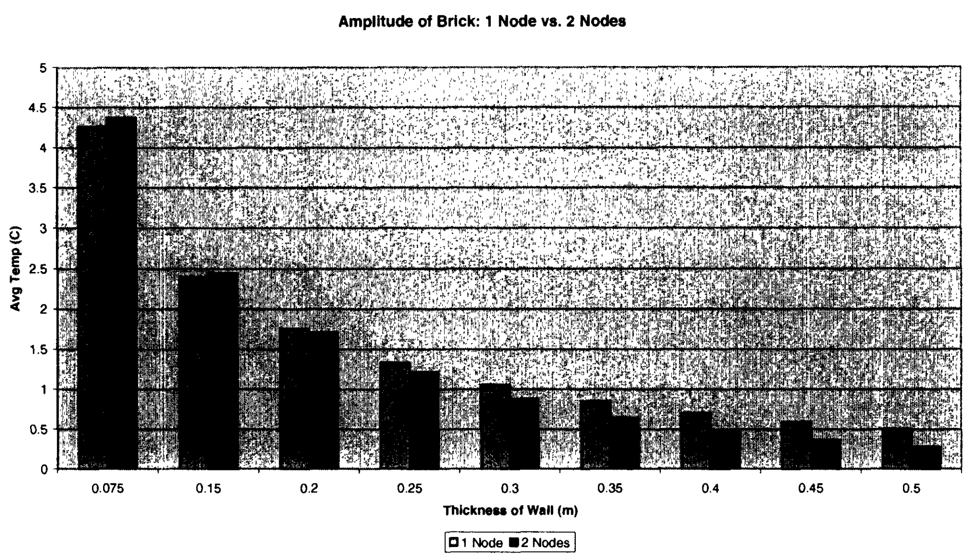


Figure 5: Amplitude of Brick with varying wall thicknesses

The amplitude of brick at 3 and 6 inches is greater than the amplitude for one-node. Somewhere between 6 and 8 inches, the amplitude for two-nodes becomes less than the amplitude for one-node and it continues to decrease with increase in wall thickness. Does the number of nodes tell another story in the behavior of certain type of materials? The data showed that concrete and brick were behaving in an unexpected way but did polystyrene and wood also behave similarly? Looking at the case of 6 and 12 inches for one-node and two-nodes said no, polystyrene and wood responded as they should, but more evidence was needed to conclude this.

The amplitude of wood for a wall thickness of 3 inches increases with the increase of nodes, but as wall thickness increases the amplitude of wood decreases with the increase of nodes. The increase between node-one and three-nodes is significant, three-nodes is about 22% less than one-node for 6 inches and at 14 inches it is 79% less than one-node. However, two-nodes and three-nodes have small changes in amplitude. Three-nodes is 2% less than two-nodes at 8 inches and 29% less than two-nodes at 14 inches, as shown in Table 9 and Figure 6. Since the difference between two- and three- nodes for wood is small, modeling the system as two nodes would give an acceptable estimate of how wood will respond.

Table 9: Amplitude of wood for given thickness and node

Wood	Node 1	Node 2	Node 3
0.075	2.548	2.584	2.659
0.150	0.947	0.718	0.736
0.200	0.587	0.346	0.338
0.250	0.398	0.183	0.167
0.300	0.287	0.105	0.086
0.350	0.217	0.064	0.045

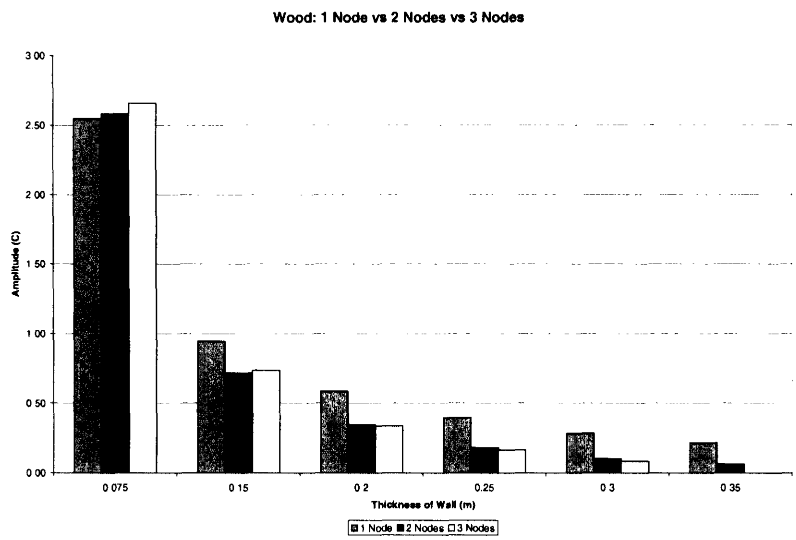


Figure 6: Amplitude of wood for given thickness and node

Table 10: Comparison of 1 node and 2 nodes to 3 nodes for wood

Wood	(Node1- Node 3)/Node 1	(Node2- Node 3)/Node 2
0.075	-4%	-3%
0.150	22%	-3%
0.200	42%	2%
0.250	58%	9%
0.300	70%	18%
0.350	79%	29%

The amplitude of polystyrene for three-nodes is larger than the amplitude of one-node for the case of a wall thickness of 3 inches as seen in Table 10. With the increase of wall thickness however, the amplitude for the case of three-nodes is less than the case of one-node. Two-nodes is sufficient to analyze the system with wall material of polystyrene. The difference between two- and three- nodes is minimal compared to the difference between one-node and three-nodes as seen in Table 11 and Figure 7 below. In the case of 3 inches, the change between two- and three-nodes is .69%, less than one percent. Table 12 shows by how much node 3 is less compared to node 1 and node 2.

Table 11: Amplitude of polystyrene for given thickness and node

Polystyrene	Node 1	Node 2	Node 3
0.075	2.057	2.003	2.076
0.150	0.679	0.436	0.433
0.200	0.405	0.191	0.175
0.250	0.268	0.095	0.075
0.300	0.189	0.052	0.034
0.350	0.141	0.031	0.016

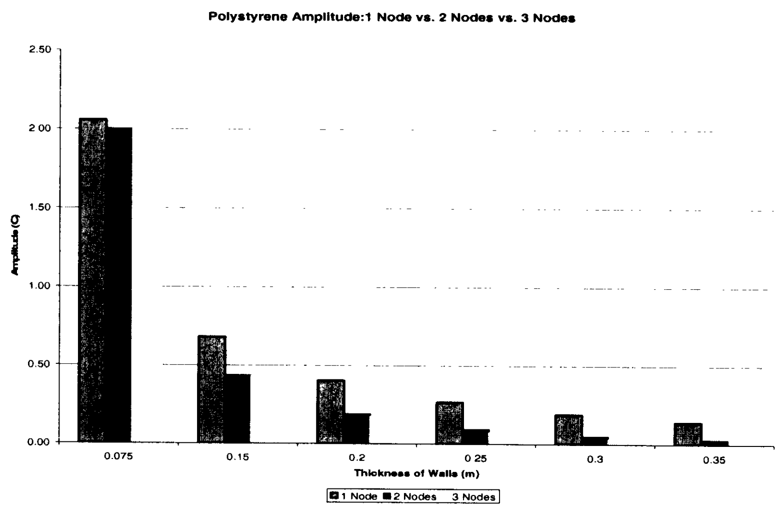


Figure 7: Amplitude of Polystyrene for given thickness and node

Table 12: Comparison of 1 node and 2 nodes to 3 nodes for polystyrene

Polystyrene	(Node1- Node 3)/Node 1	(Node2- Node 3)/Node 2
0.075	-1%	-4%
0.150	36%	1%
0.200	57%	9%
0.250	72%	21%
0.300	82%	35%
0.350	88%	47%

Wood and polystyrene at three nodes had larger amplitudes than one node for a wall 3 inches thick but a wall of 6 inches or greater, the amplitude of these two materials at three-nodes was less than the amplitude for one-node. The amplitudes of concrete and brick for three-nodes did not become less than for the case of one-node until a wall thickness of about 8 inches for brick and 10 inches for concrete. Although this seems to suggest that the larger the density, the thicker a wall has to be before the amplitude at three-nodes is less than the amplitude at one-node, the frequency plots suggest that even thicknesses of 3 and 6 inches experience higher amplitude for three-nodes than one- and two- nodes at lower frequencies. In the case of polystyrene at a thickness of 3 inches, amplitude of three- nodes is higher than one- and two- nodes for frequencies ranging from 10^-5 to 6x10^-5 [rads/sec]. For a 6 inch wall composed of polystyrene, the frequencies become even smaller ranging from 3x10^-6 to 2x10^-5[rads/sec]. This frequency range starts decreasing as thickness of the wall increases. The three-node model is simply a more accurate model.

Discussion

Knowing how a material will affect the system input is beneficial in terms of designing buildings for optimal energy savings. By knowing how a material such as concrete affects the air inside a building given a sinusoidal input, then any material similar to concrete in terms of density and specific heat will likely respond the same way. Concrete which has a phase shift ranging from 6.5 hrs to 11 hrs to 15.66 hrs for 1,2 and 3 nodes respectively, says that if the ambient temperature peaks at 1 pm, the air inside the building will not reach its peak temperature until 7:30pm for one node, 12 pm for two nodes, and 6:40am for 3 nodes. This is for a case where there are no leaks, no windows, no doors, and no radiation. The peak temperature of the building inside will not reach the same high temperature of the ambient air, it will be less. The peak amplitude of concrete for 3 inches is about 5 degrees Celsius for all nodes, where as the 14 inch case, the peak amplitude that the air inside will reach will be about 1 degree Celsius, that is 6 degrees less than the average ambient temperature when it peaks. Recall that the input average temperature (amplitude) of ambient air is 7.5 degrees Celsius. A material like polystyrene blocks most of the temperature from going through the wall and reaching the air inside the building. At 14 inches, polystyrene has amplitude of about .14, .03, and .016 for 1, 2, and 3 nodes respectively, so there is greater phase shift which translates to very small amplitude. Polystyrene is a good insulator, just like it keeps temperature from going into the building, it also keeps temperature from going out of the building, so the inside air temperature will see small changes of temperature.

Conclusion

The number of nodes that best describes a system depends on the type of material and also on the thickness of the wall. This study explored how nodal modeling influenced the amplitude of interior temperature at its peak as well as the time delay for inside air to peak. The thicker the wall the more delay there is for the interior air to peak. There are greater increases in time delay with an increase in wall thickness and density. Better insulating material which have higher thermal resistance that are less dense are best modeled with two-nodes for thicknesses of 3-6 inches and for 10-14 inches three-nodes is reasonable.

Concrete and brick, which have higher densities than polystyrene and wood, would best be described by one- and two- nodes. Since the difference between one- and two- nodes is not great one can choose either depending on the thickness of the wall as well. The two-node model for concrete and brick are for the thicker sized walls. Polystyrene is best described with two-nodes for 3 to 6 inch walls and three-nodes for thicker walls. Wood, which has a higher thermal conductivity and is less dense than polystyrene, would be best described with one-node for a 3 inch wall, two-nodes for 6 to 10 inches, and three-nodes for 14 inches.

If this study continues, the next step will be to include windows, radiation, and air-flow in the system and repeat the same type of analysis to determine how many nodes is enough. Future work can also consists of exploring what the benefits of thermal mass are compared to the problems associated with it. How does thermal mass affect the indoor air temperature and how does that vary with changes in the thickness of the floor slabs. Determining which floor slab thicknesses are and are not important as well as understanding how air flow affects night slab cool down are other areas to explore.

REFERENCES

Watlow Electric Manufacturing Company. "Physical Properties of solids, liquids, gases, Properties of non-metallic solids-Ref 132." St. Louis, Missouri, USA. May 05, 2005 <http://www.watlow.com/reference/files/nonmetallic.pdf>.

Appendix A: Analytical Solutions

Case 1:

Nomenclature:

R_1 = Resistances due to convection on outside wall
R_4 = Resistances due to convection on inside wall

R_2 = Resistance due to conduction
R_3 = Resistance due to conduction
h_{ci} = Coefficient of convection on inside
h_{co} = Coefficient of convection on outside
A = Surface area of building
t = Thickness of wall
C_w = Heat Capacitance of material of wall
C_a = Heat Capacitance of dry air
K = Thermal Conductivity of the material of wall
$T_{a,i}$ = Temperature of interior air
$T_{a,o}$ = Temperature of ambient air
T_w = Temperature of Wall at node 1

Where:

$$T_{au1} = (R_3 + R_4)C_a$$

$$T_{au2} = \left[\frac{(R_1 + R_2)(R_3 + R_4)}{2R + R_1 + R_4}\right]C_w$$

$$P = \frac{(R_3 + R_4)}{2R + R_1 + R_4} \qquad N = \frac{R_1 + R_2}{2R + R_1 + R_4}$$

$$R_1 = \frac{1}{h_{co}A} \qquad R_4 = \frac{1}{h_{ci}A}$$

$$R_2 = R_3 = \frac{t/2}{KA} = R$$

Solving Differential Equation

The two differential equations that describe the one node model are both in terms of the temperature in the wall which is unknown. Taking the derivative of T_w of equation 1a in respect to time gives equation 1c as the result.

$$T_{au1}\frac{dT_{a,i}}{dt} + T_{a,i} = T_w \qquad \text{(1a)}$$

$$T_{au2}\frac{dT_w}{dt} + T_w = PT_{a,o} + NT_{a,i} \qquad (1b)$$

$$\frac{dT_w}{dt} = T_{au1}\frac{d^2T_{a,i}}{dt^2} + \frac{dT_{a,i}}{dt} \qquad (1c)$$

To have a single equation in terms of $T_{a,i}$ and $T_{a,o}$ substitute equation 1c into 1b and equation 1a into 1b. The result is equation 1d, which is in terms of temperature of air outside and inside.

$$T_{au1}T_{au2}\frac{d^2T_{a,i}}{dt^2} + T_{au2}\frac{dT_{a,i}}{dt} + T_{au2}\frac{dT_{a,i}}{dt} + T_{a,i} = PT_{a,o} + NT_{a,i} \qquad (1d)$$

$$T_{au1}T_{au2}\frac{d^2T_{a,i}}{dt} + (T_{au1} + T_{au2})\frac{dT_{a,i}}{dt} + (1-N)T_{a,i} = PT_{a,o} \qquad (1e)$$

To further solve equation 1e the Laplace Transform of equation 1e is taken. The following equations show the steps taken to get to the solution.

$$(T_{au1}T_{au2})s^2T_{a,i}(s) + sT_{a,i}(o) - T_{a,i}(o) + (T_{au1} + T_{au2})sT_{a,i}(s) - T_{a,i}(o) + (1-N)T_{a,i}(s) = PT_{a,o}$$

$$T_{a,i}(s)\left((T_{au1} + T_{au2})s^2 + (T_{au1} + T_{au2})s + (1-N)\right) = PT_{a,o}$$

Case 2:

Nomenclature:

R_1 = Resistance due to convection on outside
R_6 = Resistance due to convection on inside
R = Resistance due to conduction in wall
h_{ci} = Coefficient of convection on inside
h_{co} = Coefficient of convection on outside
A = Surface area of building
t = Thickness of wall
C_w = Heat Capacitance of material of wall
C_a = Heat Capacitance of dry air
K = Thermal Conductivity of the material of wall
T_{w1} = Temperature of Wall at node 1
T_{w2} Temperature of Wall at node 2
$T_{a,i}$ = Temperature of interior air
$T_{a,o}$ = Temperature of ambient air

There are three unknowns and three equations. The final differential equation should be in terms of $T_{a,o}$ and $T_{a,i}$.

$$R_1 = \frac{1}{h_{c,o}A}; \quad R_6 = \frac{1}{h_{c,i}A} \text{ and } R=R_2=R_3=R_4=R_5=\frac{1}{4}\left(\frac{t}{kA}\right).$$

The following equations are rearranged:

$$(R_5 + R_6)C_a \frac{dT_{a,i}}{dt} + T_{a,i} = Tw_2 \tag{2a}$$

$$\left(\frac{(R_5+R_6)(R_3+R_4)}{R_3+R_4+R_5+R_6}\right)\frac{C_w}{2}\frac{dT_{w2}}{dt} + T_{w2} = \frac{(R_5+R_6)}{(R_3+R_4+R_5+R_6)}T_{w1} + \frac{(R_3+R_4)}{(R_3+R_4+R_5+R_6)}T_{a,i} \tag{2b}$$

$$\left(\frac{(R_1+R_2)(R_3+R_4)}{R_3+R_4+R_2+R_1}\right)\frac{C_w}{2}\frac{dT_{w1}}{dt} + T_{w1} = \frac{(R_1+R_2)}{(R_3+R_4+R_1+R_2)}T_{w2} + \frac{(R_3+R_4)}{(R_3+R_4+R_1+R_2)}T_{a,o} \tag{2c}$$

Where:

$$T_{au1} = (R_5+R_6)C_a \qquad T_{au2} = \left(\frac{(R_5+R_6)(R_3+R_4)}{R_3+R_4+R_5+R_6}\right)\frac{C_w}{2}$$

$$T_{au3} = \left(\frac{(R_1+R_2)(R_3+R_4)}{R_3+R_4+R_2+R_1}\right)\frac{C_w}{2}$$

$$D = \frac{(R_3+R_4)}{(R_3+R_4+R_1+R_2)} \quad \text{and} \quad E = \frac{(R_1+R_2)}{(R_3+R_4+R_1+R_2)}$$

$$M = \frac{(R_5+R_6)}{(R_3+R_4+R_5+R_6)} \quad \text{and} \quad L = \frac{(R_3+R_4)}{(R_3+R_4+R_5+R_6)}$$

Differential equations become:

$$T_{au1}\frac{dT_{a,i}}{dt} + T_{a,i} = T_{w2} \tag{2d}$$

$$T_{au2}\frac{dT_{w2}}{dt} + T_{w2} = MT_{w1} + LT_{a,i} \tag{2e}$$

$$T_{au3}\frac{dT_{w1}}{dt} + T_{w1} = DT_{a,o} + ET_{w2} \tag{2f}$$

Case 3:

Nomenclature:

R_1 = Resistances due to convection on outside wall

R_6 = Resistances due to convection on inside wall

$R_2 = R_3$ = Resistance due to conduction

$R_4 = R_5$ = Resistance due to conduction

h_{ci} = Coefficient of convection on inside

h_{co} = Coefficient of convection on outside

A = Surface area for corresponding material

t = Thickness of wall

C_w = Heat Capacitance of material of wall
C_a = Heat Capacitance of dry air
K_1 = Thermal Conductivity of wall material one
K_2 = Thermal Conductivity of wall material two
T_1 = Temperature of Wall at node 1
T_2 = Temperature of Wall at node 2
$T_{a,i}$ = Temperature of interior air
$T_{a,o}$ = Temperature of ambient air
Where:

$$R_2 = R_3 = \frac{\frac{1}{3}t}{K_1 A_1}$$

$$R_4 = R_5 = \frac{\frac{1}{6}t}{K_2 A_2}$$

Expanding the differential Equations gives the following solution:

$$(R_5 + R_6)C_a \frac{dT_{a,i}}{dt} = T_2 - T_{a,i} \qquad (3a)$$

$$\left[\frac{(R_3 + R_4)(R_5 + R_6)}{R_3 + R_4 + R_5 + R_6}\right] C_2 \frac{dT_2}{dt} = \frac{(R_3 + R_4)}{(R_3 + R_4 + R_5 + R_6)} T_{a,i} + \frac{(R_5 + R_6)}{(R_3 + R_4 + R_5 + R_6)} T_1 - T_2 \qquad (3b)$$

$$\left[\frac{(R_1 + R_2)(R_3 + R_4)}{(R_1 + R_2 + R_3 + R_4)}\right] C_1 \frac{dT_1}{dt} = \frac{(R_1 + R_2)}{(R_1 + R_2 + R_3 + R_4)} T_2 + \frac{(R_3 + R_4)}{(R_1 + R_2 + R_3 + R_4)} T_{a,o} - T_1 \qquad (3c)$$

Case 4:
Nomenclature:

C_a =heat capacitance of air
C=heat capacitance of wall material
$T_{a,o}$ =temperature of air outside
$T_{a,i}$ =temperature of air inside
T_1 =temperature at node 1
T_2 =temperature at node 2
T_3 =temperature at node 3

R_1 =resistance due to convection on outside
R_8 =resistance due to convection on inside
R_2 =R_3 =R_4 =R_5 =R_6 =R_7 =resistance due to convection

$$R_1 = \frac{1}{h_{c,o}A}; R = \frac{1}{h_{c,i}A}; R_2 = R_3 = R_4 = R_5 = R_6 = R_7 = \frac{\frac{1}{6}t}{K}$$

Solving the differential equations gives the following:

$$\frac{dT_{a,i}}{dt} = -\frac{1}{T_{au1}} T_{a,i} + \frac{1}{T_{au1}} T_3 \quad \text{(4a)}$$

$$\frac{dT_3}{dt} = -\frac{1}{T_{au2}} T_3 + \frac{1}{T_{au2}\left[1+\left(\frac{R_7+R_8}{R_5+R_6}\right)\right]} T_{a,i} + \frac{1}{T_{au2}\left[1+\left(\frac{R_5+R_6}{R_7+R_8}\right)\right]} T_2 \quad \text{(4b)}$$

$$\frac{dT_2}{dt} = -\frac{1}{T_{au3}} T_2 + \frac{1}{T_{au3}\left[1+\left(\frac{R_3+R_4}{R_5+R_6}\right)\right]} T_1 + \frac{1}{T_{au3}\left[1+\left(\frac{R_5+R_6}{R_3+R_4}\right)\right]} T_3 \quad \text{(4c)}$$

$$\frac{dT_1}{dt} = -\frac{1}{T_{au4}} T_1 + \frac{(R_1+R_2)}{T_{au4}(R_1+R_2+R_3+R_4)} T_2 + \frac{(R_3+R_4)}{T_{au4}(R_1+R_4)} T_{a,o} \quad \text{(4d)}$$

where,

$$\mathrm{T}_{au1} = (R_7+R_8)C_a \qquad \mathrm{T}_{au2} = \frac{(R_7+R_8)(R_5+R_6)}{3(R_5+R_6+R_7+R_8)} C$$

$$\mathrm{T}_{au3} = \frac{(R_3+R_4)(R_5+R_6)}{3(R_3+R_4+R_5+R_6)} C \qquad \mathrm{T}_{au4} = \frac{(R_3+R_4)(R_1+R_2)}{3(R_1+R_2+R_3+R4)} C \quad \text{(4e)}$$

Appendix B : Matlab Solutions

Matlab program was used to solve the differential equations and produce frequency plots.
Case 1:

$$N=\frac{R_3+R_4}{R_1+R_2+R_3+R_4} \qquad P=\frac{R_1+R_2}{R_1+R_2+R_3+R_4}$$

$$T_{au1}=(R_3+R_4)C_a \qquad T_{au2}=\frac{(R_1+R_2)(R_3+R_4)}{(R_1+R_2+R_3+R_4)}C_w$$

Case 2:

where:

$$T_{au1}=(R_5+R_6)C_a \qquad T_{au2}=\left[\frac{(R_3+R_4)(R_5+R_6)}{(R_3+R_4+R_5+R_6)}\right]\frac{C_w}{2}$$

$$T_{au3}=\left[\frac{(R_1+R_2)(R_3+R_4)}{(R_1+R_2+R_3+R_4)}\right]\frac{C_w}{2}$$

$$N=\frac{(R_5+R_6)}{(R_3+R_4+R_5+R_6)}; \qquad Q=\frac{(R_3+R_4)}{(R_1+R_2+R_3+R_4)}$$

$$P=\frac{(R_3+R_4)}{(R_3+R_4+R_5+R_6)}\;; \qquad S=\frac{(R_1+R_2)}{(R_1+R_2+R_3+R_4)}$$

Case 3:

Where:

$$A=\frac{1}{R_5+R_6};\; B=\frac{1}{R_3+R_4};\; C=\frac{1}{R_1+R_2};\; D=(A+B);\; E=(B+C)$$

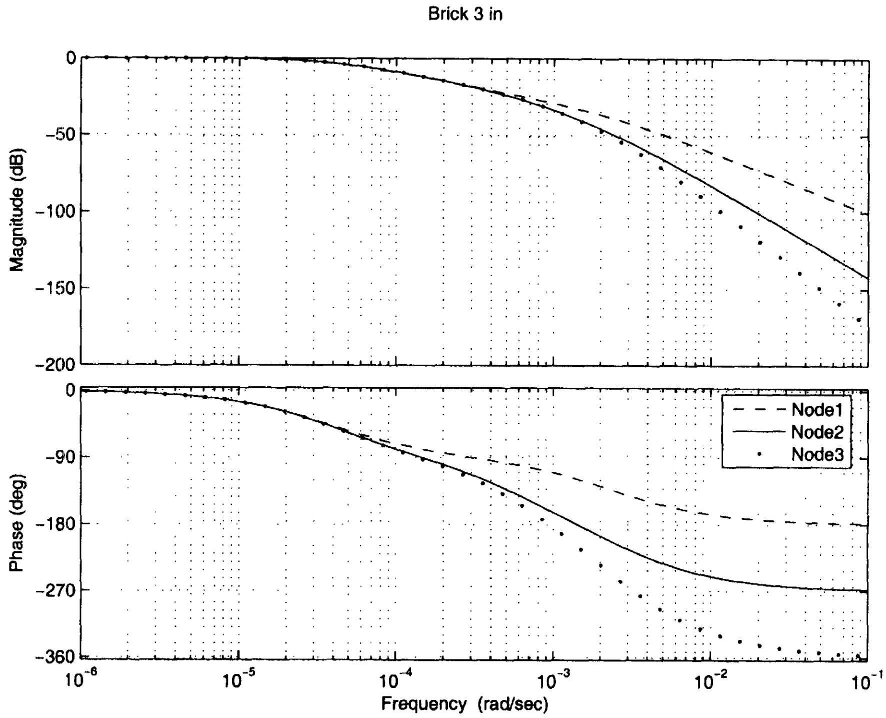
Case 4:

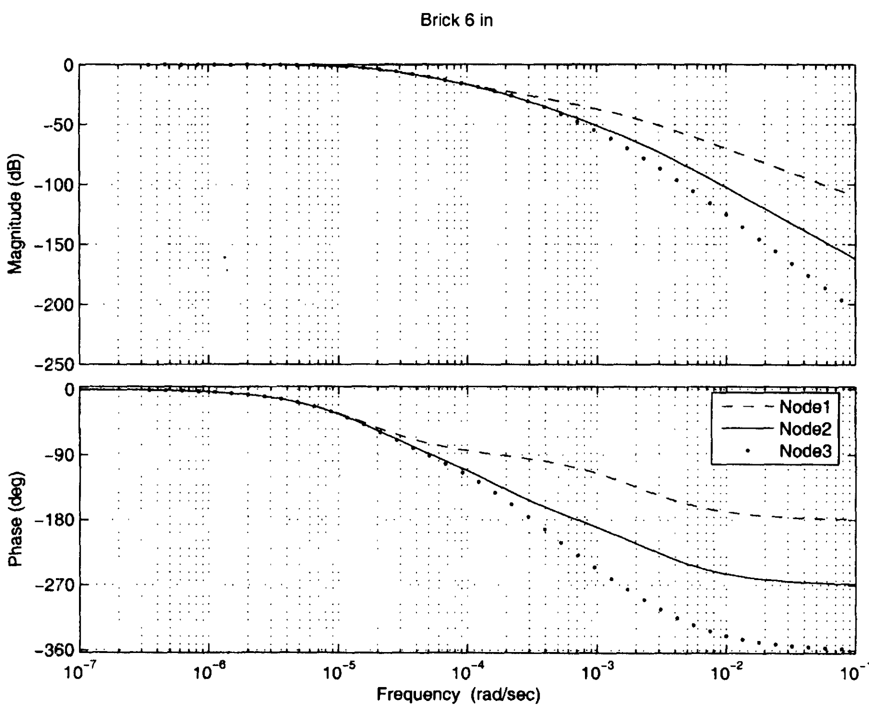
$$P=\left[1+\left(\frac{R_7+R_8}{R_5+R_6}\right)\right] \quad M=\left[\frac{R_1+R_2}{R_1+R_2+R_3+R_4}\right] \quad S=\frac{(R_3+R_4)}{R_1+R_2+R_3+R_4}$$

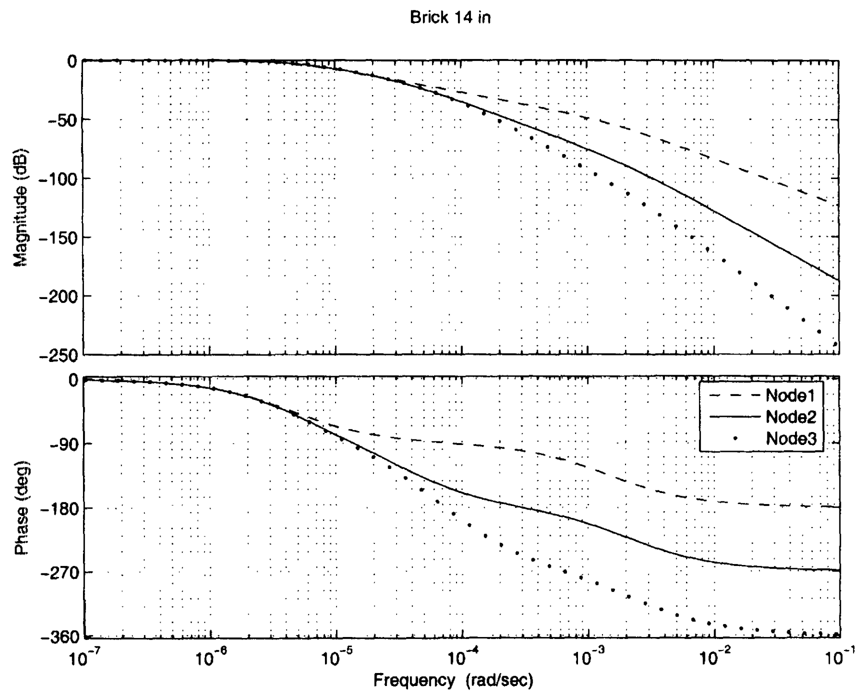
$$N=\left[1+\left(\frac{R_5+R_6}{R_7+R_8}\right)\right] \quad X=\left[1+\left(\frac{R_5+R_6}{R_3+R_4}\right)\right] \quad Q=\left[1+\left(\frac{R_7+R_4}{R_5+R_6}\right)\right]$$

Appendix C : Frequency Figures

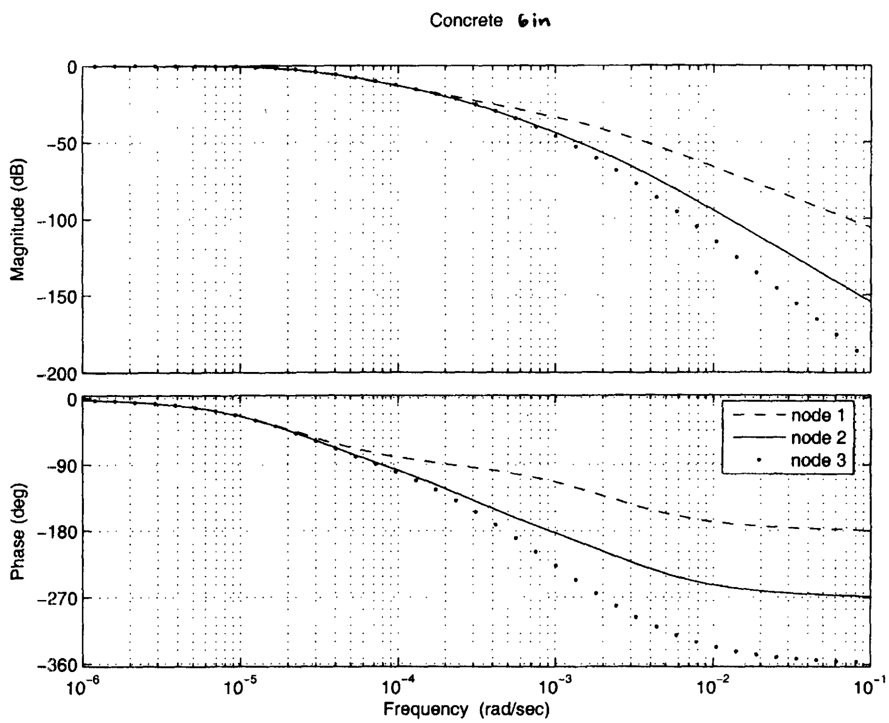
Brick:

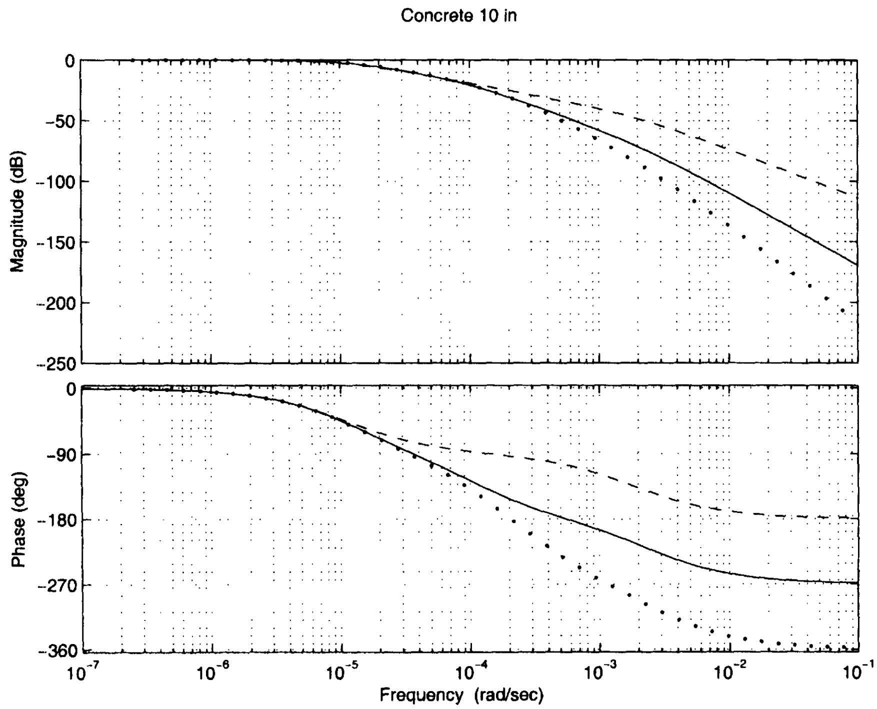


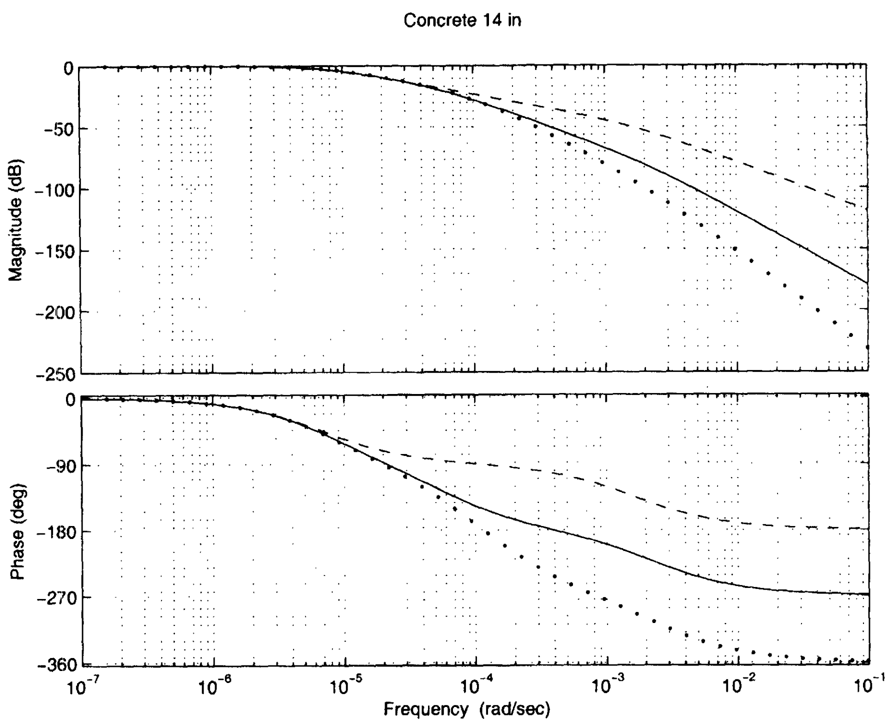



Brick 14 in
Magnitude (dB)
Phase (deg)
Node1
Node2
Node3
Frequency (rad/sec)

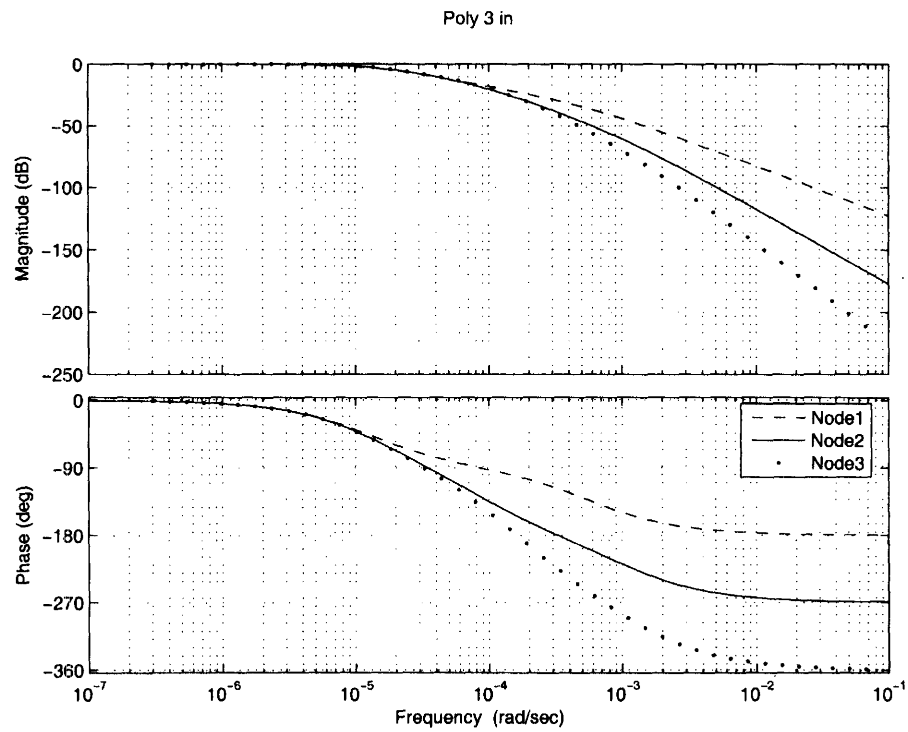
Concrete:


Concrete 6in
Magnitude (dB)
Phase (deg)
node 1
node 2
node 3
Frequency (rad/sec)

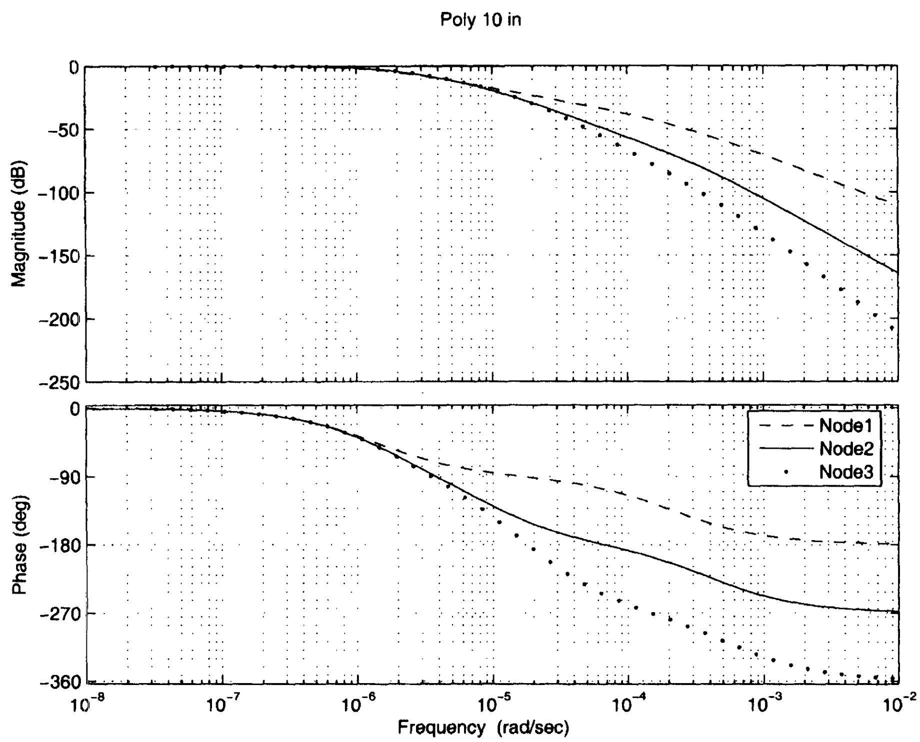

Concrete 10 in
Magnitude (dB)
0
-50
-100
-150
-200
-250
Phase (deg)
0
-90
-180
-270
-360
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
Frequency (rad/sec)


Concrete 14 in
Magnitude (dB)
0
-50
-100
-150
-200
-250
Phase (deg)
0
-90
-180
-270
-360
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
Frequency (rad/sec)

Poly:

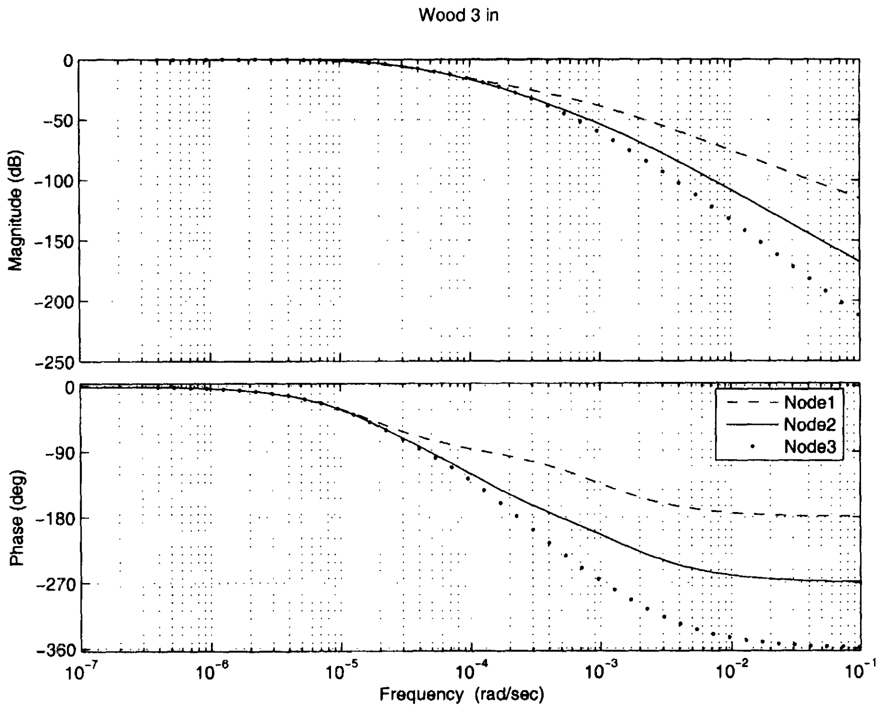


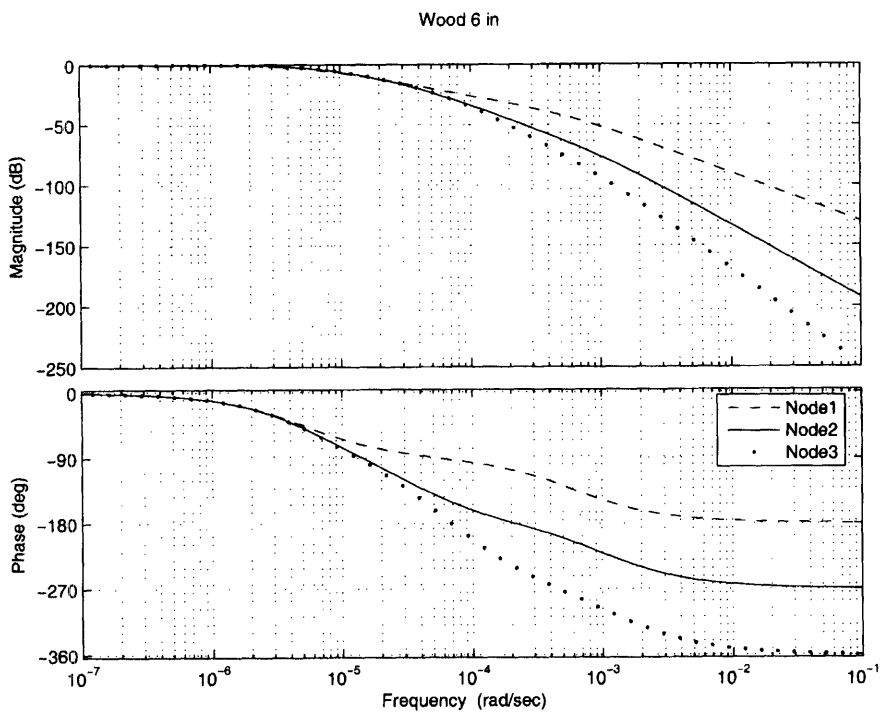
Poly 3 in
Magnitude (dB)
0
-50
-100
-150
-200
-250
Phase (deg)
0
-90
-180
-270
-360
Node1
Node2
Node3
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
Frequency (rad/sec)



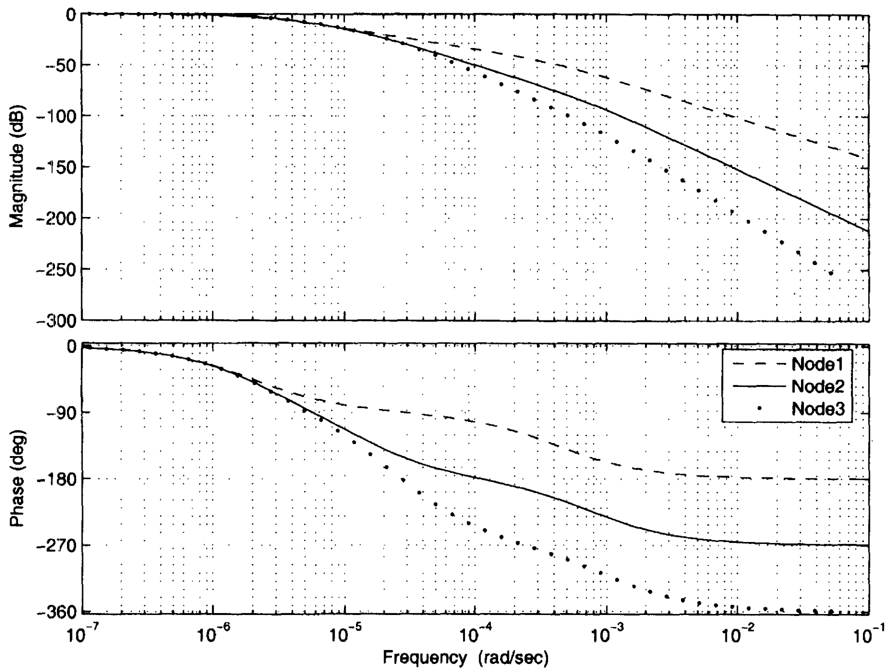
Poly 10 in
Magnitude (dB)
0
-50
-100
-150
-200
-250
Phase (deg)
0
-90
-180
-270
-360
Node1
Node2
Node3
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
Frequency (rad/sec)

Wood:

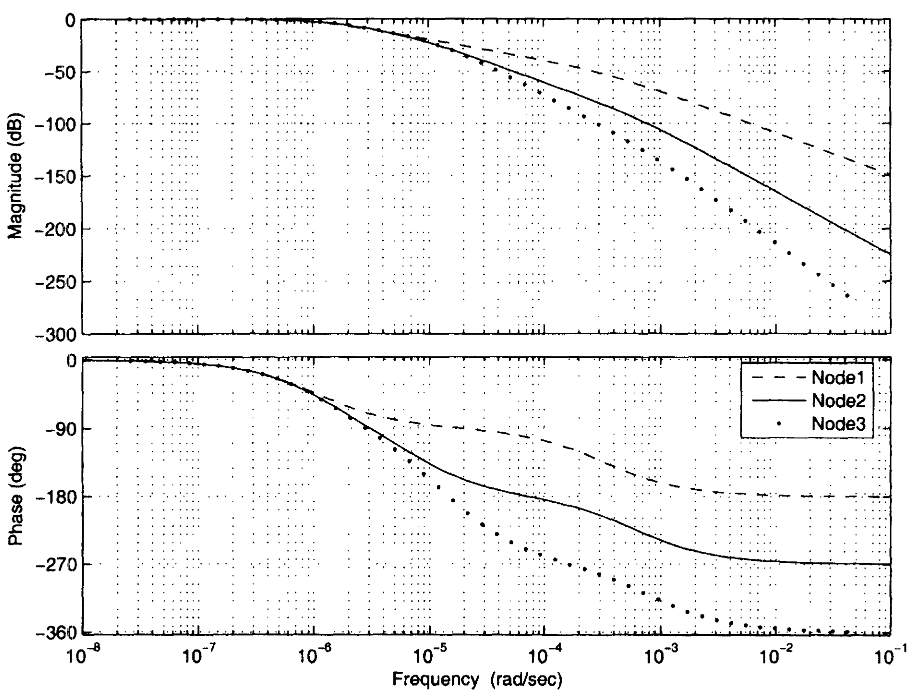

Wood 3 in
Magnitude (dB)
Phase (deg)
Node1
Node2
Node3
Frequency (rad/sec)


Wood 6 in
Magnitude (dB)
Phase (deg)
Node1
Node2
Node3
Frequency (rad/sec)

Wood 10 in



Wood 14 in



Appendix D: Matlab m. Files

Matlab file for obtaining magnitude and phase frequency plots for 1,2,3 nodes. The material properties as well as wall thickness need to be changed manually.

```
%%1,2,3 Nodes Frequency Comparison

th= .15;             % thickness of the wall in meters

L1= 30*12*2.5/100;
W1= 100*12*2.5/100;
H1= 30*12*2.5/100;
L2= L1-(2.*th);

W2= W1- (2.*th);
H2= H1- (2.*th);

%%Volumes
V1= L1*W1*H1;
V2= L2*W2*H2;
V3= V1-V2;
%%densities
pa= 1.29; % kg/m3
ppoly=1057;

%%masses
ma= pa.*V2;
mpoly= ppoly.*V3;

% thermal conductivity of wall material
Kpoly=.08;

%SPECIFIC HEAT
Cppoly=1340;


%%%NODE 1

A= 2*(L1.*H1)+3*(W1.*H1);
hci= 10;
hco= 10;
R1= 1./(hco.*A);

R2= (th./2)./(Kpoly.*A);
R3= (th./2)./(Kpoly.*A);
R= (th./2)./(Kpoly.*A);
R4= 1./(hci.*A);
Ca= (1006.*ma);
%%%%%%%%%%%%%%%%%%%%%
Cwpoly= (Cppoly.*mpoly);
%%%%%%%%%%%%%%%%%%%%%

Tau1= (R3 + R4).*Ca
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Tau2= ((R1 + R2).*(R3 + R4)./((2.*R) + R1 + R4)).*Cwpoly;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
P= (R3 + R4)./((2.*R) + R1 + R4);
N= (R1 + R2)./(R2 + R3 + R1 + R4);

a=[ (-1./Tau1) (1./Tau1); (N./Tau2) (-1./Tau2)];
b=[0 ;(P./Tau2)];
c=[1 0];
d=0;
tperiodic=(0:0.1:24*3600);
Tavg= 0;
Tamp= 7.5; %degrees C

Tao= -Tavg + Tamp*sin(2*pi*tperiodic/(3600*24));
u= Tao;

sys=ss(a,b,c,d);

figure(4);
bode(sys,'--');
title('Response to Outside Temp, Poly 6 in');
hold on
grid

%%%%%%%% NODE 2

R2= (th./4)./(Kpoly.*A);
R3= (th./4)./(Kpoly.*A);
R4= (th./4)./(Kpoly.*A);
R5= (th./4)./(Kpoly.*A);
R= (th./4)./(Kpoly.*A);
R6= 1./(hci.*A);
Ca= (1006.*ma);
Cwpoly= (Cppoly.*mpoly);

Tau1= (R5 + R6).*Ca;
Tau2= [((R.*R) + (R6.*R))./((3.*R) + R6)].*Cwpoly;
Tau3= [((R.*R1)+(R.*R))./((3*R)+R1)].*Cwpoly;
M= (R+R6)./((3*R)+R6);
L= (2.*R)./((3*R)+R6);
D= (2.*R)./((3.*R)+R1);
E= (R1 +R)./((3.*R) +R1);

a=[ (-1./Tau1) (1./Tau1) 0; (L./Tau2) (-1./Tau2) (M/Tau2); 0 (E/Tau3) (-1/Tau3)];
b=[0 ;0;(D./Tau3)];
c=[1 0 0];
d=0;
```

```
tperiodic=(0:0.1:86400);
Tavg= 0;
Tamp= 7.5; %degrees C

Tao= -Tavg + Tamp*sin(2*pi*tperiodic/86400);
u= Tao;

sys=ss(a,b,c,d);
figure(4)
bode(sys)
grid
hold on

%%%%%  NODE 3

R8= 1./(hci.*A);
R2= (1/6)*th/(Kpoly*A);
R3=(1/6)*th/(Kpoly*A);
R4=(1/6)*th/(Kpoly*A);
R5=(1/6)*th/(Kpoly*A);
R6=(1/6)*th/(Kpoly*A);
R7=(1/6)*th/(Kpoly*A);

%%Capacitance

Ca= (1006.*ma);      %J/kg C       %Capacitance of Air C=specific heat * mass
Cwpoly= (Cppoly.*mpoly);      %J/kg C       %Capacitance of the wall material

%% TAUS

Tau1= (R7 + R8)*Ca;
Tau2= ((R7 + R8)*(R5+R6)/(3*(R5+R6+R7+R8)))*Cwpoly;
Tau3= ((R3+R4)*(R5+R6)/(3*(R3+R4+R5+R6)))*Cwpoly;
Tau4= ((R3+R4)*(R1+R2)/(3*(R3+R4+R1+R2)))*Cwpoly;

P= [ 1 + ((R7+R8)/(R5+R6))];
N= [1 + ((R5+R6)/(R7+R8))];
X= [1 + ((R5+R6)/(R3+R4))];
Q= [1 + ((R3+R4)/(R5+R6))];
M= (R1+R2)/(R1 + R2+ R3+ R4);
S= (R3+R4)/(R1+R2+R3+R4);

a=[ (-1./Tau1) (1./Tau1) 0 0; (1./(Tau2.*P)) (-1./Tau2) (1./(Tau2.*N)) 0; 0 (1./
(Tau3.*X)) (-1./Tau3) (1./(Tau3.*Q)); 0 0 (M./Tau4) (-1./Tau4)];
b=[0; 0; 0;(S./Tau4)];
c=[1 0 0 0];
d=0;
tperiodic=(0:0.1:24*3600);
```

```
Tavg= 0;
Tamp= 7.5; %degrees C

Tao= -Tavg + Tamp*sin(2*pi*tperiodic/(3600*24));
u= Tao;
sys=ss(a,b,c,d);

figure(4);
bode(sys,'.');
title('Poly 6 in');
grid
hold off
```

Matlab file for phase diagrams. The following matlab file is for one-node and the example of concrete.

```
%%ONE NODE

th= .15;          % thickness of the wall in meters
L1= 30*12*2.5/100;
W1= 100*12*2.5/100;
H1= 30*12*2.5/100;
L2= L1-(2.*th);                %remember to change the m (of the whatever material) for Cw

W2= W1- (2.*th);
H2= H1- (2.*th);

%%Volumes
V1= L1*W1*H1;
V2= L2*W2*H2;
V3= V1-V2;
%%densities
pa= 1.29; % kg/m3
pcon= 2307;% kg/m3
pwood= 801;
pbrick= 2243;
ppoly=1057;

%%masses
ma= pa.*V2;
mcon= pcon.*V3;
mwood= pwood.*V3;
mbrick= pbrick.*V3;
mpoly= ppoly.*V3;

Kcon= 1.37;  %W/mC          % thermal conductivity of wall material
Kwood=.17;
Kbrick=1.15;
Kpoly=.08;

A= 2*(L1.*H1)+3*(W1.*H1);          % Total Surface Area in sqaured meters
hci= 10;  %W/(m*m) K       % coefficient of convection of air inside
hco= 10;          % hco is the coefficient of convection of the air outside
R1= 1./(hco.*A);  %Aw is the cross-sectional area of the wall
              %R1 is the resistance due to convection outside
R2= (th./2)./(Kwood.*A); %resistance due to conductance
R3= (th./2)./(Kwood.*A);
R= (th./2)./(Kwood.*A);  %where R2=R3=R
R4= 1./(hci.*A);   %resistance due to convection inside
Ca= (1006.*ma);    %J/kg C     %Capacitance of Air C=specific heat * mass
Cwwood= (2387.*mwood);    %J/kg C    %Capacitance of the wall material
%Tai   %Temperature of the air inside in units of C
%Tao        %Temperature of the air outside in units of C
%Tw          %Temperature of the wall in units of C
```

```
Tau1= (R3 + R4).*Ca
Tau2= ((R1 + R2).*(R3 + R4)./((2.*R) + R1 + R4)).*Cwwood;
P= (R3 + R4)./((2.*R) + R1 + R4);
N= (R1 + R2)./(R2 + R3 + R1 + R4);

a=[ (-1./Tau1) (1./Tau1); (N./Tau2) (-1./Tau2)];
b=[0 ;(P./Tau2)];
c=[1 0];
d=0;
tperiodic=(0:0.1:24*3600);
Tavg= 0;
Tamp= 7.5; %degrees C

Tao= -Tavg + Tamp*sin(2*pi*tperiodic/(3600*24));
u= Tao;

sys=ss(a,b,c,d);
[TairesponsetoTao]=lsim(sys,u,tperiodic);
figure(1)
plot(tperiodic,TairesponsetoTao,'-.',tperiodic,u,'b');
max(TairesponsetoTao)
xlabel('Time(sec)');
ylabel('Tair (C)');
title('Response to Outside Temp, Wood 6 in');
legend('Tai', 'Tao');
Axis([0 86400 -7.5 7.5]);
grid

figure(2);
bode(sys);
title('Response to Outside Temp, Concrete 6 in');
grid

%%Polystyrene
R2= (th./2)./(Kpoly.*A); %resistance due to conductance
R3= (th./2)./(Kpoly.*A);
R= (th./2)./(Kpoly.*A);  %where R2=R3=R

Cwpoly= (1340.*mpoly);
Tau1= (R3 + R4).*Ca;
Tau2= ((R1 + R2).*(R3 + R4)./((2.*R) + R1 + R4)).*Cwpoly;
a=[ (-1./Tau1) (1./Tau1); (N./Tau2) (-1./Tau2)];
b=[0 ;(P./Tau2)];
c=[1 0];
d=0;
tperiodic=(0:0.1:24*3600);
Tavg= 0;
Tamp= 7.5; %degrees C
```

```
Tao= -Tavg + Tamp*sin(2*pi*tperiodic/(3600*24));
u= Tao;

sys=ss(a,b,c,d);
[TairesponsetoTao]=lsim(sys,u,tperiodic);
figure(5)
plot(tperiodic,TairesponsetoTao,'-.',tperiodic,u,'b');
max(TairesponsetoTao)
xlabel('Time(sec)');
ylabel('Tair (C)');
title('Response to Outside Temp, Polystyrene 6 in');
legend('Tai', 'Tao');
Axis([0 86400 -7.5 7.5]);
grid


figure(6)
bode(sys);
title('Response to Outside Temp, Polystyrene 6 in');
grid

%%Concrete

Kcon= 1.37;  %W/mC        % thermal conductivity of wall material
Kwood=.17;
Kbrick=1.15;
Kpoly=.08;

A= 2*(L1.*H1)+3*(W1.*H1);          % Total Surface Area in sqaured meters
hci= 10;  %W/(m*m) K      % coefficient of convection of air inside
hco= 10;         % hco is the coefficient of convection of the air outside
R1= 1./(hco.*A);  %Aw is the cross-sectional area of the wall
            %R1 is the resistance due to convection outside
R2= (th./2)./(Kcon.*A); %resistance due to conductance
R3= (th./2)./(Kcon.*A);
R= (th./2)./(Kcon.*A);  %where R2=R3=R
R4= 1./(hci.*A);  %resistance due to convection inside
Ca= (1006.*ma);    %J/kg C    %Capacitance of Air C=specific heat * mass
Cwcon= (653.*mcon);    %J/kg C    %Capacitance of the wall material
%Tai   %Temperature of the air inside in units of C
%Tao        %Temperature of the air outside in units of C
%Tw          %Temperature of the wall in units of C
Tau1= (R3 + R4).*Ca
Tau2= ((R1 + R2).*(R3 + R4)./((2.*R) + R1 + R4)).*Cwcon;
P= (R3 + R4)./((2.*R) + R1 + R4);
N= (R1 + R2)./(R2 + R3 + R1 + R4);
```

```
a=[ (-1./Tau1) (1./Tau1); (N./Tau2) (-1./Tau2)];
b=[0 ;(P./Tau2)];
c=[1 0];
d=0;
tperiodic=(0:0.1:24*3600);
Tavg= 0;
Tamp= 7.5; %degrees C

Tao= -Tavg + Tamp*sin(2*pi*tperiodic/(3600*24));
u= Tao;

sys=ss(a,b,c,d);
[TairesponsetoTao]=lsim(sys,u,tperiodic);
figure(1)
plot(tperiodic,TairesponsetoTao,'-.',tperiodic,u,'b');
max(TairesponsetoTao)
xlabel('Time(sec)');
ylabel('Tair (C)');
title('Response to Outside Temp, Concrete 6 in');
legend('Tai', 'Tao');
Axis([0 86400 -7.5 7.5]);
grid

figure(2);
bode(sys);
title('Response to Outside Temp, Concrete 6 in');
grid

%%Brick
R2= (th./2)./(Kbrick.*A); %resistance due to conductance
R3= (th./2)./(Kbrick.*A);
R= (th./2)./(Kbrick.*A); %where R2=R3=R

Cwb= (921.*mbrick);
Tau1= (R3 + R4).*Ca;
Tau2= ((R1 + R2).*(R3 + R4)./((2.*R) + R1 + R4)).*Cwb;
a=[ (-1./Tau1) (1./Tau1); (N./Tau2) (-1./Tau2)];
b=[0 ;(P./Tau2)];
c=[1 0];
d=0;
tperiodic=(0:0.1:24*3600);
Tavg= 0;
Tamp= 7.5; %degrees C

Tao= -Tavg + Tamp*sin(2*pi*tperiodic/(3600*24));
u= Tao;

sys=ss(a,b,c,d);
```

```
[TairesponsetoTao]=lsim(sys,u,tperiodic);
figure(5)
plot(tperiodic,TairesponsetoTao,'-.',tperiodic,u,'b');
max(TairesponsetoTao)
xlabel('Time(sec)');
ylabel('Tair (C)');
title('Response to Outside Temp, Brick 6 in');
legend('Tai', 'Tao');
Axis([0 86400 -7.5 7.5]);
grid


figure(6)
bode(sys);
title('Response to Outside Temp, Brick 6 in');
grid
```

The following is a matlab file for two-node case of one material only.

```
%%TWO NODES

Kcon= 1.37;  %W/mC        % thermal conductivity of wall material
Kwood=.17;
Kbrick=1.15;
Kpoly=.08;


%%CONCRETE

th= .15;   % thickness of wall

L1= 30*12*2.5/100;
W1= 100*12*2.5/100;
H1= 30*12*2.5/100;          %remember to change the mas (of the whatever material) for
Cw
L2= L1-(2.*th);
W2= W1- (2.*th);
H2= H1- (2.*th);

%%Volumes
V1= L1*W1*H1;
V2= L2*W2*H2;
V3= V1-V2;
%%densities
pa= 1.29; % kg/m3
pcon= 2307;% kg/m3
pwood= 801;
pbrick= 2243;
ppoly=1057;

%%masses
ma= pa.*V2;
mcon= pcon.*V3;
mwood= pwood.*V3;
mbrick= pbrick.*V3;
mpoly= ppoly.*V3;

A= 2*(L1.*H1)+3*(W1.*H1);          % Total Surface Area in sqaured meters
hci= 10;  %W/(m*m) K        % coefficient of convection of air inside
hco= 10;          % hco is the coefficient of convection of the air outside
R1= 1./(hco.*A);  %Aw is the cross-sectional area of the wall
             %R1 is the resistance due to convection outside
R2= (th./4)./(Kcon.*A); %resistance due to conductance
R3= (th./4)./(Kcon.*A);
R4= (th./4)./(Kcon.*A);
R5= (th./4)./(Kcon.*A);
R= (th./4)./(Kcon.*A);  %where R= R2=R3=R4=R5
```

```
R6= 1./(hci.*A);   %resistance due to convection inside
Ca= (1006.*ma);    %J/kg C     %Capacitance of Air C=specific heat * mass
Cwcon= (653.*mcon);     %J/kg C     %Capacitance of the wall material
%Tai    %Temperature of the air inside in units of C
%Tao          %Temperature of the air outside in units of C
%Tw           %Temperature of the wall in units of C
Tau1= (R5 + R6).*Ca;
Tau2= [((R.*R) + (R6.*R))./((3.*R) + R6)].*Cwcon;
Tau3= [((R.*R1)+(R.*R))./((3*R)+R1)].*Cwcon;
M= (R+R6)./((3*R)+R6);
L= (2.*R)./((3*R)+R6);
D= (2.*R)./((3.*R)+R1);
E= (R1 +R)./((3.*R) +R1);

a=[ (-1./Tau1) (1./Tau1) 0; (L./Tau2) (-1./Tau2) (M/Tau2); 0 (E/Tau3) (-1/Tau3)];
b=[0 ;0;(D./Tau3)];
c=[1 0 0];
d=0;
tperiodic=(0:0.1:86400);
Tavg= 0;
Tamp= 7.5; %degrees C

Tao= -Tavg + Tamp*sin(2*pi*tperiodic/86400);
u= Tao;

sys=ss(a,b,c,d);
[TairesponsetoTao]=lsim(sys,u,tperiodic);
figure(1)
plot(tperiodic,TairesponsetoTao,'-.',tperiodic,u,'b');
xlabel('Time(sec)');
ylabel('Tair (C)');
title('Response to Outside Temp, Concrete 6 in');
legend('Tai', 'Tao');
Axis([0 86400 -7.5 7.5]);
grid

figure(2)
bode(sys)
title('Concrete 6 in');
grid
```

The following is a matlab file for three-node case for only one material.

```
%% 3 Nodes

th= .15;          % thickness of the wall in meters
L1= 30*12*2.5/100;
W1= 100*12*2.5/100;
H1= 30*12*2.5/100;
L2= L1-(2.*th);             %remember to change the m (of the whatever material) for Cw

W2= W1- (2.*th);
H2= H1- (2.*th);

%%Volumes
V1= L1*W1*H1;
V2= L2*W2*H2;
V3= V1-V2;
%%densities
pa= 1.29; % kg/m3
pcon= 2307;% kg/m3
pwood= 801;
pbrick= 2243;
ppoly=1057;

%%masses
ma= pa.*V2;
mcon= pcon.*V3;
mwood= pwood.*V3;
mbrick= pbrick.*V3;
mpoly= ppoly.*V3;
Kcon= 1.37;  %W/mC         % thermal conductivity of wall material
Kwood=.17;
Kbrick=1.15;
Kpoly=.08;

A= 2*(L1.*H1)+3*(W1.*H1);          % Total Surface Area in sqaured meters
hci= 10;  %W/(m*m) K      % coefficient of convection of air inside
hco= 10;         % hco is the coefficient of convection of the air outside
R1= 1./(hco.*A);  %Aw is the cross-sectional area of the wall
           %R1 is the resistance due to convection outside
R8= 1./(hci.*A);


%%Concrete
R2= (1/6)*th/(Kcon*A);
R3=(1/6)*th/(Kcon*A);
R4=(1/6)*th/(Kcon*A);
R5=(1/6)*th/(Kcon*A);
R6=(1/6)*th/(Kcon*A);
R7=(1/6)*th/(Kcon*A);
```

```
%%Capacitance

Ca= (1006.*ma);     %J/kg C      %Capacitance of Air C=specific heat * mass
Ccon= (653.*mcon);     %J/kg C      %Capacitance of the wall material

%% TAUS

Tau1= (R7 + R8)*Ca;
Tau2= ((R7 + R8)*(R5+R6)/(3*(R5+R6+R7+R8)))*Ccon;
Tau3= ((R3+R4)*(R5+R6)/(3*(R3+R4+R5+R6)))*Ccon;
Tau4= ((R3+R4)*(R1+R2)/(3*(R3+R4+R1+R2)))*Ccon;

P= [ 1 + ((R7+R8)/(R5+R6))];
N= [1 + ((R5+R6)/(R7+R8))];
X= [1 + ((R5+R6)/(R3+R4))];
Q= [1 + ((R3+R4)/(R5+R6))];
M= (R1+R2)/(R1 + R2+ R3+ R4);
S= (R3+R4)/(R1+R2+R3+R4);

a=[ (-1./Tau1) (1./Tau1) 0 0; (1./(Tau2.*P)) (-1./Tau2) (1./(Tau2.*N)) 0; 0 (1./(Tau3.*X))
(-1./Tau3) (1./(Tau3.*Q)); 0 0 (M./Tau4) (-1./Tau4)];
b=[0; 0; 0;(S./Tau4)];
c=[1 0 0 0];
d=0;
tperiodic=(0:0.1:24*3600);
Tavg= 0;
Tamp= 7.5; %degrees C

Tao= -Tavg + Tamp*sin(2*pi*tperiodic/(3600*24));
u= Tao;

sys=ss(a,b,c,d);
[TairesponsetoTao]=lsim(sys,u,tperiodic);
figure(1)
plot(tperiodic,TairesponsetoTao,'-.',tperiodic,u,'b');
max(TairesponsetoTao)
xlabel('Time(sec)');
ylabel('Tair (C)');
title('Response to Outside Temp, Concrete 6 in');
legend('Tai', 'Tao');
Axis([0 86400 -7.5 7.5]);
grid

figure(2);
bode(sys);
title('Response to Outside Temp, Concrete 6 in');
grid
```